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Optimal Design of Education System: Theory and Evidence*

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Abstract

This paper studies the optimal design of an economy's education system, where education serves to develop specific skills for students and sort them into two tracks—academic and vocational—based on their ability levels. Using data from OECD countries, we first examine international differences in education systems and labor market outcomes, focusing on income inequality and skill mismatches. We identify some facts: 1) higher government spending on vocational education is linked to lower income inequality and skill mismatches, 2) greater participation in academic education correlates with more skill mismatches and higher inequality.

We then construct a theoretical model where a social planner designs an education system aimed at balancing economic output and income inequality. The model includes individuals with varying ability levels, firms in service and manufacturing sectors, and a social planner who sets education investments and academic quotas. Education tracks both develop skills and signal abilities to employers, influencing job placements. As more students enroll in the academic track, the model shows that labor market returns to academic education decline, leading to skill mismatches and greater inequality. The planner faces a tradeoff between limiting academic education to improve efficiency or expanding it to reduce inequality.

Several extensions to the model are considered, including the effects of inverted wage gaps, refined education sorting within the academic track, and reforms that integrate education tracks or adjust the timing of student sorting. Additionally, we explore the policy implications of our theory for education-system reforms, such as the optimal level of investment in vocational education and the quota for academic education, showing how these decisions influence income inequality and efficiency.

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1 Introduction

The relationship between education and labor market outcomes has evolved significantly, particularly in the context of diploma inflation and skill mismatches. While education is often seen as a pathway to a better life, many students find that their degrees no longer guarantee access to good jobs. Enrollment expansion has led to diploma inflation, where the increasing number of people with higher education degrees diminishes the value of those qualifications in the job market. As a result, individuals holding diplomas or bachelor's degrees are competing for jobs that may not require such credentials, which devalues their educational achievements. This, coupled with skill mismatches, creates a situation where individuals are overeducated but lack the specific skills employers need. Educational mismatches occur when workers' educational attainment is inconsistent with job requirements, leaving some overeducated for their roles and others undereducated.

Income inequality has risen dramatically in recent decades, becoming a central topic in public debate. Education choices also contribute to this issue, as individuals often prefer academic over vocational education due to societal perceptions and systemic factors. In countries like China, for example, the expansion of higher education since 1999 has led to a decline in vocational school enrollment, as more students opt for general high schools. This trend is exacerbated by underdeveloped vocational education systems that fail to align with employer needs, resulting in lower social recognition and reduced income for vocational graduates. Internationally, countries approach education and labor market integration differently. In nations like Germany, Austria, and the Nordic countries, where early sorting into vocational and academic tracks is common, there are lower levels of income inequality and fewer skill mismatches compared to countries that delay or avoid such tracking. These differences highlight the varied impact that education systems can have on labor market outcomes and income distribution.

The evidence discussed above clearly indicates that a country's education system significantly affects labor market outcomes. However, most existing research focuses primarily on the impact of educational tracking and de-tracking on students' academic achievements, with less attention given to the long-term labor market effects. This paper aims to fill that gap by addressing several key research questions: How are different education systems related to labor market outcomes? How can we establish a clear linkage between education systems and labor market performance? And how should we interpret the variations in education systems across nations, particularly in light of trends such as diploma inflation and de-tracking?[1]

To address these questions, we begin by presenting empirical evidence that highlights international differences in vocational education, income inequality, and skill mismatches among OECD countries. Following this, we introduce a theoretical framework that incorporates a social planner's design of the education system, the educational sorting of students into various tracks, and the labor market sorting of individuals into job positions. This theory provides an equilibrium outcome that aligns with the observed evidence and offers insights into the optimal design of education systems to better align educational and labor market outcomes.

We first provide evidence on international differences in vocational education, income inequality, and skill mismatches among OECD countries. Our findings highlight two key patterns: First, higher government spending per capita on vocational education is linked to lower skill mismatches and income inequality. Countries investing more in vocational training tend to experience better alignment between worker skills and job requirements and more equitable income distribution. Second, higher participation rates in academic education correlate with greater skill mismatches and higher income inequality. These trends emphasize the

[1] De-tracking involves higher participation in vocational education and delayed age at which students are sorted into educational tracks.

role of education system structures in shaping labor market outcomes.

Motivated by facts discussed above, we then construct a parsimonious yet rich theoretical framework that generates equilibrium behavior consistent with these facts. Specifically, we consider an economy consisting of three types of players: a mass one of individuals with heterogeneous ability levels, a mass one of job positions/firms, and a social planner who manages the economy's education system, which consists of two education tracks: an $a$cademic track and a $v$ocational track. Each individual is endowed with a *non-observable* innate ability $\theta \in [0,1]$, which reflects the individual's ability ranking in the population—an individual with ability level θ is higher ability than individuals with ability levels falling in the range $[0,\theta)$. Education serves two main purposes: 1) developing specific skills for students; 2) selecting students into the two education tracks based on their ability levels. Concerning the design of education system, the social planner can *invest* in education so that each vocational (resp. academic) track student acquires specific skills h (resp. $\tilde{h}$); the social planner can also impose a *quota* $Q \in [0,1]$ for academic education, i.e., at most Q individuals can receive academic education. The labor market consists of two sectors of firms: a $s$ervice (resp. $m$anufacture) sector with a mass of $q \in (0,1)$ (resp. $1-q$), where service sector productions can better leverage worker ability than manufacture sector productions but manufacture sector productions require some specific skills. As education selects students into the two tracks based upon their ability levels, it also serves as a signal of ability when students enter the labor market. So, individuals are sorted into the two sectors based upon their education attainments. Whenever there are more individuals with an equivalent education attainment than the number of job positions available from a sector, we assume that individuals are sorted into that sector on a *random* basis.

An equilibrium $\mathcal{E} = \{E;\lambda;W\}$ consists of the social planner's choice of education system $E = \{Q,h,\tilde{h}\}$, the enrollment level λ for academic education which aggregates each individual's education choice, and the wage structure $W = \{w_{ij}\}$, where w_{ij} denotes the wage to a worker received education $i \in \{a,v\}$ and employed by sector $j \in \{s,m\}$. One interesting aspect of our analysis is that the labor-market returns W and individuals' education choices are co-determined in equilibrium. In particular, the return to academic education decreases with the enrollment level λ for academic education. The logic is that, when λ increases or more individuals receive academic education, academic education, though positively signals an individual's ability, serves as a weaker signal, because the average ability of academic track students is equal to $1-\frac{\lambda}{2}$; moreover, due to random matching discussed above, an academic student can be employed by the service sector with probability $\frac{q}{\lambda}$, which declines with λ.[2]

To develop model intuitions, we begin with the baseline analysis which is confined to equilibrium behavior when the social planner can only invest in vocational education, i.e., $h > 0$ but $\tilde{h} = 0$. Define $\Lambda(h)$, with $\partial\Lambda(h)/\partial h < 0$, as the demand for academic education which equates the labor-market return to the two education tracks. The quota Q imposed by the social planner means a limited supply of academic education. In equilibrium, if $Q \geq \Lambda(h)$ or there is an excess supply for academic education, then at most $\Lambda(h)$ individuals receive academic education. In contrast, if $Q < \Lambda(h)$ or there is an excess demand for academic education, then the education system features *education tracking*, where at most Q individuals can be admitted into academic education and the other $\Lambda(h) - Q$ individuals who demand academic education are unwillingly separated into vocational education. Most of our analyses focus on equilibrium behavior when there is an excess demand, i.e., $\Lambda(h) \geq q$.

Regarding the design of education system, we assume that the government aims to not only maximize the

[2] One can interpret this equilibrium outcome as *diploma inflation*: with a surplus of highly educated people, workers with higher education move downward to take jobs that do not match their education.

economy's overall output (i.e., efficiency) but also minimize income inequality (i.e., equity). In particular, we use $\gamma \in (0,1)$ to capture the importance of equity, and the average wage gap between the high-paying sector and the low-paying sector to measure the degree of income inequality. We establish that the social planner faces an *equity-efficiency* tradeoff in the choice of the quota Q for academic education, because a small quota, though increases the overall output, widens the wage gap.

The logic is as follows. Consider a social planner with $\gamma \to 0$ who just centers on efficiency. Then, the social planner should impose $Q = q$, which suggests that the equilibrium enrollment level is $\lambda = q$ (because of the excess demand for academic education $\Lambda(h) \geq q$) and, in turn, each individual with academic (resp. vocational) education is employed as a service (resp. manufacture) sector worker. In this case, the social planner achieves an *efficient* outcome, where there is no skill mismatch—no manufacture sector worker lacks specific skills. Note, however, that this efficient outcome is associated with a high wage for the service sector, i.e., $1-\frac{\lambda}{2} = 1-\frac{q}{2}$ when $\lambda = q$, which results in a wide wage gap between the high-paying sector (i.e., service) and the low-paying sector (i.e., service). Thus, to reduce income inequality, a social planner that cares very much about equity will have incentives to increase the quota Q to an excessively high level, i.e., $Q > q$, because a large quota means that more individuals can now receive academic education (i.e., $Q \uparrow \Longrightarrow \lambda \uparrow$) and, in turn, decreases the wage $1-\frac{\lambda}{2}$ to the high-paying sector. In this case, the social planner achieves an *inefficient* outcome where there are skill mismatches—some academic track students lacking specific skills are employed as manufacture-sector workers.

We show that there exists a threshold value γ^* concerning the importance of equity, such that the social planner imposes a quota $Q > q$ whenever $\gamma > \gamma^*$. This equilibrium behavior can be envisaged as an *expansion* of academic education, where the social planner chooses an inefficiently high quota. Our theory rationalizes why it is of the social planner's interest to expand academic education. That is, typically a more natural approach to reduce income inequality is for the social planner to improve the amount of specific skills acquired by vocational track students by investing heavily in vocational education, which increases the wage to the manufacture sector and, in turn, reduces the wage gap. However, when the investment cost is too high, the wage to the manufacture sector is relatively too low, which requires the social planner to take further actions to reduce income inequality. According to our analysis, a large quota for academic education or an expansion of academic education, though entails diploma inflation, serves to decrease the wage to the service sector which, in turn, narrows the wage gap across the two sectors.

We then explore three enriched analyses concerning the design of education system. To better understand the essence of equilibrium behavior, we twist the baseline model in one direction at a time. In the first extension, we study equilibrium behavior when there is an excess supply or low demand for academic education, i.e., $\Lambda(h) \leq q$. In this case, there is an *inverted* wage gap in equilibrium, where the high-paying (resp. low-paying) sector is the manufacture (resp. service) sector. Moreover, the low demand for academic education corresponds to a high demand for vocational education, so the social planner should now impose a quota $\tilde{Q}$ on vocational education. As opposed to the baseline analysis, the social planner does not face an equity-efficiency tradeoff. This is because an increased λ or a higher enrollment level for academic education not only increases efficiency but also narrows the wage gap. The logic is that an increase in λ means that more academic track students who are higher ability ones than vocational track students can be sorted into the service sector which better leverages ability. In turn, there is improved productive efficiency and an increased wage to the low-paying sector (i.e., service). Hence, regardless of the importance of equity, the social planner always imposes a quota $Q = q$, which suggests $\lambda = q$ as the equilibrium enrollment level for academic education and, in turn, no skill mismatches.

In the second extension, we study equilibrium behavior when there is a refined sorting of academic track students. This analysis is motivated by real-world observations that within academic track, outstanding students—those with high ability levels—can be admitted into elite schools or further pursue advanced degrees. Specifically, as opposed to the baseline setup where there is just one type of schooling outcome within academic education, we now assume that students are sorted into two tiers according to their ability levels. In turn, when $\Lambda(h) \geq q$ or there is an excess demand for academic education, the process of selecting individuals into the service is no longer random; instead, tier 1 students have better chances to be employed than tier 2 students. In this case, as was true for the baseline analysis, we show that the social planner faces an equity-efficiency tradeoff in the choice of the quota Q for academic education. In equilibrium, there also exists a threshold value concerning the importance of equity, such that the social planner imposes a quota $Q > q$ whenever the importance of equity exceeds the threshold. The threshold value is now lower than the counterpart from the baseline analysis, indicating that the planner is now less likely to expand academic education to reduce income inequality. The logic is that in terms of inequality reduction, expanding academic education now becomes less effective, because it impacts on the wage to tier 2 students but not on that to tier 1 students. Yet, whenever an expansion of academic education is employed to reduce inequality, the degree of expansion is higher than that from the baseline analysis due to its decreased effectiveness just discussed.

In the third extension, we examine two types of education-system reforms. 1) We consider whether the social planner should integrate the two education tracks, where students from either track can acquire specific skills. We call this reform as integrated tracks because teaching contents/materials across the two tracks become more aligned in terms of developing skills specific to the manufacture sector. 2) We study the optimal timing of education sorting, where the social planner additionally decides when to sort students into the two tracks. Basically, we show that the baseline analysis' logic continues to apply to these two reforms. That is, the social planner still faces an equity-efficiency tradeoff in the choice of the quota Q for academic education. As detailed below, the new results here concern the reform's effect on the quota or the degree of expansion for academic education.

Specifically, when the social planner can invest $\tilde{h} > 0$ and $h > 0$, respectively, in academic track and vocational track, the social planner is more likely to expand academic education than in the baseline analysis. The logic is that while narrowing the wage gap, an expansion given integrated tracks results in a smaller efficiency loss due to skill mismatches than that from the baseline analysis, because academic track students can now also acquire specific skills $\tilde{h} > 0$ which prepare them for manufacture sector productions when not being selected into the service sector. This provides the social planner with more opportunities to reduce inequality via an expansion of academic education. Also, $\tilde{h} > 0$ improves the labor-market return to academic education and, in turn, yields a higher demand $\Lambda(h, \tilde{h})$. So, whenever the social planner expands academic education, the degree of expansion is larger than that from the baseline analysis, i.e., $\Lambda(h, \tilde{h}) > \Lambda(h, 0)$ for any $h > 0$ and $\tilde{h} > 0$.

As for the optimal timing of education sorting, in contrast to the baseline analysis where the sorting is perfectly ability based, we now assume that the extent to which education sorting is ability based is determined by the timing of sorting. In particular, deferring the sorting to a later stage of student career means that the sorting will be more based upon an individual's ability. In this case, the social planner faces a tradeoff: a deferred sorting means that an individual's schooling choice serves as a stronger signal of ability, yet a declining marginal return to investing in vocational education because there will be less time left for vocational track students to acquire specific skills. We show that the social planner's optimal design of education system again depends on the importance of equity. In particular, if equity is important or $\gamma \geq q$, the social planner should force the sorting to take place as early as possible, which reduces the signaling role of education and, in turn, decreases inequality. As for the quota, the social planner impose a quota $Q > q$ (resp. $Q = q$) if the importance of equity is sufficiently high (resp. otherwise). On the contrary, if efficiency is important or $q > \gamma$, then the social planner should impose a quota $Q = q$, and defer the sorting to a time point that optimally balances the two effects. In particular, as the importance of efficiency grows, the social planner should further defer the sorting of student to a later stage, which improves the effectiveness of ability sorting and, in turn, increases productive efficiency.

Equilibrium behavior of the baseline model is consistent with aforementioned stylized facts. That is, an decrease in academic education participation rate and/or an increase in vocational education investment contribute to not only a lesser extent of skill mismatches but also a smaller degree income inequality. Note, in particular, that in most countries, the enrollment level for academic education satisfies $\lambda \geq q$. So, when matching the theory to the data, our focus is on the baseline analysis where there is an excess demand for academic education, i.e., $\Lambda(h) \geq q$. We first consider facts concerning academic education participation rates. Recall that the social planner expand academic education (via imposing a large quota $Q > q$) to reduce inequality. In equilibrium, the extent of expansion depends on the degree of income inequality, where a wider wage gap would induce more students to receive academic education. Thus, we obtain a positive relationship between academic education participation rates and income inequality. Moreover, we find a positive relationship between academic education participation rates and the degree of skill mismatches, which is a byproduct of expanding academic education. Next, consider facts concerning vocational education investments. According to our analysis, an increased vocational education investment improves vocational track students' specific skills, which attract more students into vocational education and increase the wage to the low-paying sector. In turn, we find that vocational education spendings negatively correlate with the degree of skill mismatches and income inequality.

Contributions

To our best knowledge, this paper is the first to study the optimal design of education system from a social planner's perspective, with a focus on an education system's impact on the sorting of individuals into both education tracks as well as job positions. As detailed in Section **??**, most existing studies center on *micro* level effects of education systems, e.g., the short-term effects of education (de)tracking on educational outcomes and inter-generational mobility concerning human capital and income, or the long-term effects of education (de)tracking on individuals' labor-market outcomes. Yet, relatively less is known about the education system's impact on an economy's labor-market outcomes at the *macro* level. To this end, this paper, instead, examine the impact of education systems on the sorting of individuals into both education tracks as well as job positions. In doing so, we first provide an empirical analysis, followed by a theoretical model that yields equilibrium behavior consistent with the data.

More specifically, our theory combines several conventional views of education from the economics literature. Basically, education plays two roles: one is skill development to prepare students for labor-market productions (Becker, 1962, 1964; Ben-Porath, 1967), and the other is ability sorting (screening) of individuals (Spence, 1973; Farber and Gibbons, 1996; Altonji and Pierret, 2001; Lange, 2007; Kahn and Lange, 2014); in turn, an individual's education choice serves as a signal of ability in the labor market.[3,4] In studying the design of education system, other than the choice of education spendings on skill development, one novel

[3] note: this is also related to statistical discrimination (a hiring process based on education attainment)

[4] There is also a literature on the effects of education on an individual's promotion prospects in the labor market. See, for instance, Bernhardt (1995), DeVaro and Waldman (2012), and Waldman (2016).

aspect of our analysis is that the social planner can impose a quota that limits the number of students for academic education. Analogous to many classical models in public economics, as discussed above, the social planner faces an equity-efficiency tradeoff in the choice of the quota for academic education, because a small quota, though increases efficiency, widens the wage gap. In equilibrium, given a high importance of equity or narrowing the wage inequality, the social planner has incentives to choose an inefficiently large quota. In this regard, our theory provides a *novel* explanation for observations that educations expansions and diploma inflation are prevalent in many countries. So, the theory improves our understanding of the basic economic forces for determining an economy's education system, and sheds new light on related real-world education policies and reforms.

Last but least, our theory contributes to the literature on assortative matching with information frictions (Roth and Xing, 1994; Li and Suen, 2000; Damiano, Li, and Suen, 2005). Putting in the labor-market context, when there is complementarity between worker types and firm types, an efficient outcome requires that higher ability individuals are matched with more productive firms, i.e., a positive sorting, which might be hindered by various types of information frictions, e.g., belated information, private information, costly information acquisitions, etc. This paper departs from the literature by considering a two-stage process, where individuals are first sorted into two education tracks based on their ability levels and then selected into job positions according to their education attainments. As discussed above, due to the equity concern, the social planner sometimes favors an inefficient outcome, where the sorting is non-positive. To this end, academic education is expanded to coarsen the information content of academic education and, in turn, narrow the wage gap. So, as opposed to existing studies' focus on whether information frictions can hinder positive sorting, coarsened information is part of the design of optimal education system, where an imperfect education sorting and its ensued inefficient outcome is optimal from the social planner's perspective.

The remainder of the paper is structured as follows. Section 2 demonstrates stylized facts. Section 3 describes the baseline model, which is analyzed in Section 4. Section 5 considers various extensions of the baseline model. Section 6 discuss the theory's empirical evidence as well as related policy recommendations and education reforms. Section 7 concludes. All technical details are relegated to the appendix.

2 Stylized Facts

In this section, we first summarize the institutional background regarding education tracking, then demonstrate several stylized facts to motivate as empirical evidence to support the model setting.

2.1 Institutional Background: Education Tracking Systems Across Countries

Education tracking systems, which sort students into academic or vocational pathways, vary widely across countries, influencing both educational and economic outcomes. In many countries, the age at which students are tracked, the degree of formality in the system, and the long-term consequences of such tracking differ significantly.

In Germany and Austria, students are tracked into different educational pathways as early as age 10, where they are sorted into either vocational or academic tracks based on ability. This early tracking is designed to provide focused education and practical job training for vocational students, ensuring that they are prepared for specific roles in the labor market (Lovenheim and Smith (2023); Woessmann (2009)). While early tracking tends to improve labor market alignment and reduce skill mismatches, it has also been associated with reinforcing educational inequality, as students' future opportunities are determined at a young age (Woessmann (2009)).

In contrast, countries like Finland and Norway delay tracking until students are around 15 or 16 years old, offering a more flexible approach that allows students more time to explore different educational paths before making career-defining decisions. This delayed tracking can provide greater opportunities for social mobility but may present challenges in aligning education outcomes with labor market needs, as students are not directed into vocational training until later in their academic careers (Lovenheim and Smith (2023); Woessmann (2009)).

The United States employs a more informal tracking system, where students are not formally sorted into vocational or academic pathways until high school. This results in a more comprehensive system of education, giving students broader options. However, this delayed and less structured tracking system has been criticized for contributing to skill mismatches and inconsistencies in educational preparedness (Betts (2011)). Unlike the structured early tracking systems in Germany, U.S. students face more variability in the quality of their education and the readiness for specific roles in the labor market (Betts (2011)).

In Central and Eastern Europe, educational tracking has faced unique challenges due to the region's transition from communist-era education systems. Efforts to integrate these systems with Western European standards, particularly through the Bologna Process, have been gradual. The Bologna Process aims to standardize degrees and promote mobility within the European Higher Education Area, yet countries in this region have struggled to establish consistent academic and vocational tracking systems that align with these broader educational reforms (Kwiek (2004)).

In summary, education tracking systems vary widely across countries, with early tracking models like those in Germany providing more immediate labor market alignment but potentially reinforcing inequality. In contrast, delayed tracking systems in countries like Finland and Norway offer greater flexibility but may pose challenges in meeting labor market demands (Lovenheim and Smith (2023); Woessmann (2009)). The U.S., with its informal and later tracking, faces its own challenges in preparing students for the workforce (Betts (2011)), while Central and Eastern European countries continue to adapt their systems within the broader context of European integration (Kwiek (2004)).

2.2 Data sources and key variables

In this section, we demonstrate several stylized facts to motivate and provide evidence for the model setting.

2.3 Data sources and key variables

We use the ILOSTAT database collected by the International Labour Organization to calculate the employment and income related variables. The database provides annual data for over 120 countries, covering variables on population, labor force, employment, wages, and other labor related aggregate statistics. We also use GDP measure from the the OECD data. All statistics are calculated from annual panel data at country level. Monetary statistics are in 2017 U.S. dollars, and divide all monetary values by the country's GDP per capita to ensure comparability across countries. [5] Because the panel data is highly unbalanced due to missing data issue, we take the average of all statistics among available data points between 2014 and 2019, so the analysis would not be affected by the impacts of the pandemic.

[5] The ILOSTAT data coverts local currency units to US dollars using market exchange rates and also using 2017 purchasing power parities (PPPs) for private consumption. The OECD data is in USD by year.

Enrollment rate into vocational track In the annual data of working-age population by education level, we define the vocational track as upper secondary education, post-secondary non-tertiary education, and short-cycle tertiary education, and denote its population size by N_v. We define the academic track as Bachelor's or equivalent level and denote its population size by N_a.[6] Then we can define the enrollment rate into academic education track by $pr_a = \frac{N_a}{N_a+N_v}$.

Fraction of workers in service jobs In the annual data of working-age population by pccupation, we define service jobs as manager occupations and professional occupations, and denote its population size by $N_{service}$. We define manufacturing jobs as Technicians and associate professionals, Clerical support workers, Service and sales workers, Craft and related trades workers, and Plant and machine operators, and assemblers, and denote its population size by $N_{manufacturing}$.[7] Then we can define the enrollment rate into academic education track by $pr_s = \frac{N_{service}}{N_{service}+N_{manufacturing}}$.

Income inequality Combining the occupation classification into service and manufacturing jobs and average earnings by occupations, we calculate the average earnings of the two types of jobs. This statistic is a proxy for income inequality mentioned in the model as part of the government's objective.

Average labor market return to vocational education The average labor market return to vocational education is $\text{ATE} = E\left[Y_i \left| edu = vocational\right.\right] - E\left[Y_i \left| edu = 0\right.\right]$. However, we only have data on the number of working age population with vocational education or training by education levels (less than basic, basic, intermediate, or advanced), and the average hourly earnings by education levels. We first identify the education level in which a single education track splits into dual education tracks (i.e. vocational and academic) by country and year, and denote this education level by edu_{dual}. Then we approximate $E\left[Y_i \left| edu = vocational\right.\right]$ by $E\left[Y_i \left| edu = edu_{dual}\right.\right]$ and $E\left[Y_i \left| edu = 0\right.\right]$ by $E\left[Y_i \left| edu = edu_{dual} - 1\right.\right]$. This approximation makes two assumptions: (1) academic and vocational track workers whose education attainment are edu_{dual} have the same average earnings, (2) academic and vocational track workers whose education attainment are $edu_{dual} - 1$ also have the same average earnings.

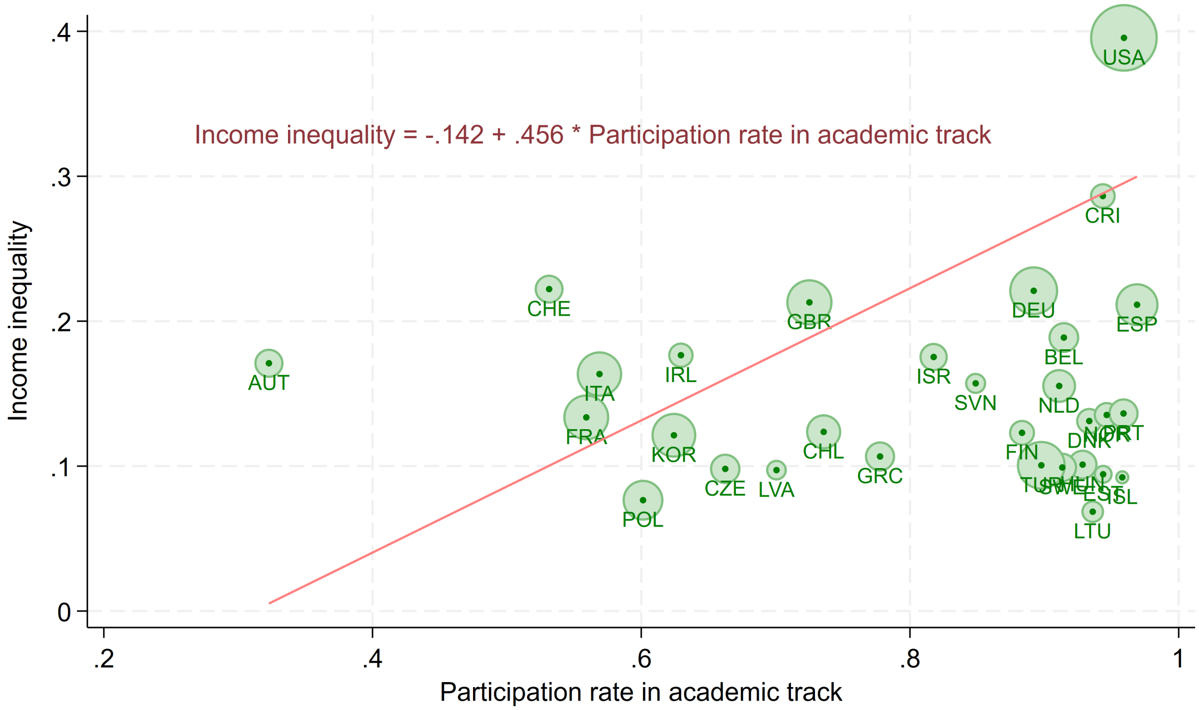
Government's investment on vocational education We extract from OECD database the government's spending on vocational education per vocational student. The statistic is in U.S. dollars and is a proxy for s in the model.

Degree of skill mismatch in the labor market Workers with academic track education background but are working in manufacturing jobs suffer from skill mismatch between labor demand and supply. Conversely, workers with vocational track and service jobs are also mismatched. We calculate the share of mismatched workers in the total working population in either job type, and use it as a proxy for the degree of skill mismatch.

[6] These four education attainment levels are relevant subpopulation for education tracking. We exclude no schooling, primary education, lower secondary education, Master's or equivalent level, or not elsewhere classified education groups from the calculation.

[7] We exclude Elementary occupations, Armed forces occupations, and Not elsewhere classified.

Figure 1: Income Inequality and Participation Rate in Academic Track



Notes: This figure shows the correlation between income inequality and participation rate in the academic track in OECD countries. Both statistics are calculated using the 2014-2019 ILOSTAT panel database. The participation rate in academic track is calculated from the total working population and the population of people with vocational training. The income inequality is the difference in average income levels of service and manufacturing jobs divided by GDP per capita. Considering the panel data has gaps and is unbalanced, we take the average of available annual data by country. The size of the circle reflects the average working population size of the country. We fit a simple regression weighted by working population size, and draw the fitted line.

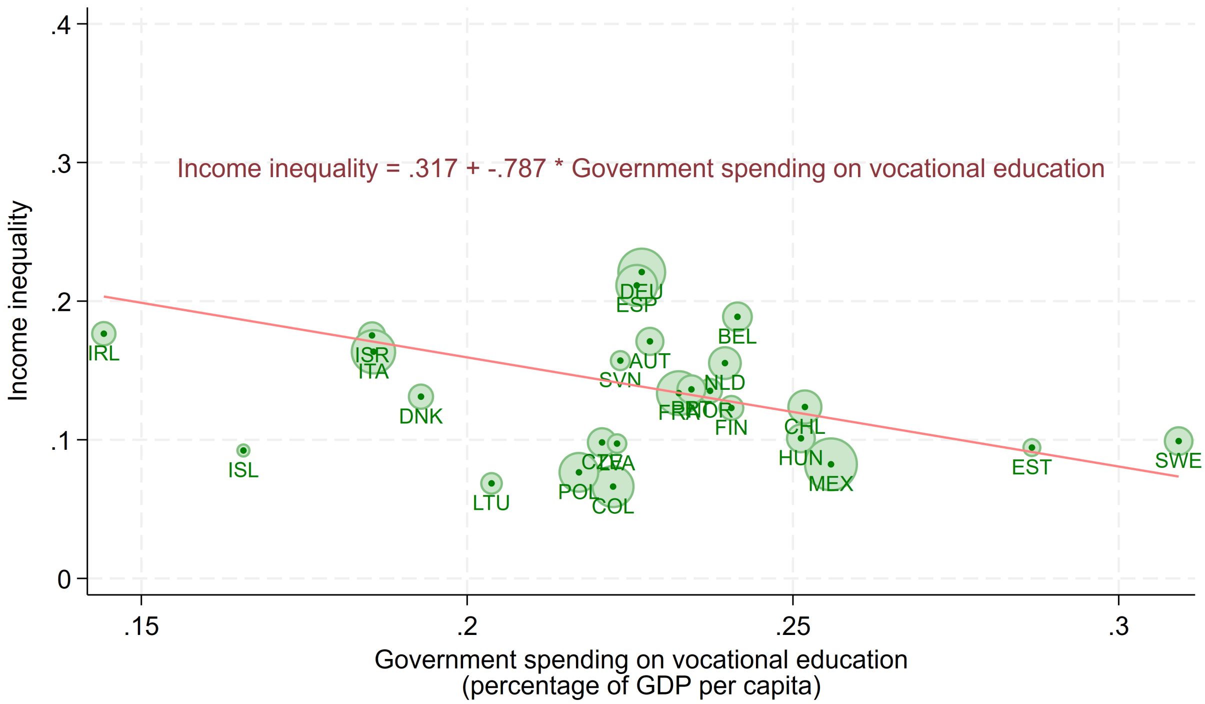
2.4 Stylized facts

Figure 1 shows the association between income inequality and the participation rate in academic education track in OECD countries. Each green dot represents a country, and the area of the circle around the dot reflects the working age population of the country. The fitted line is a simple regression weighted by population sizes.

In most OECD countries, the academic track participation rates are above 60%, with an average of 80%. Austria has the smallest academic track participation rate of 32.3%. Income inequality, measured by the average income gap between service and manufacturing jobs as a percentage of the country's GDP per capita, ranges between 6.6% for Lithuania and 39.5% for the U.S. The relationship between the academic track participation rate and income inequality is overall positive. A simple linear regression gives a slope of 0.456, meaning that each 10% increase in the education tracking into academic tracks would widen the average income gap between service and manufacturing jobs by 4.56% of the GDP per capita of the respective country.

Figure 2 shows the association between the government spending on vocational education and income inequality. Government spending on vocational education, measured by the spending on each student divided by the country's GDP per capita, ranges between 8.2% (Ireland) and 30.9% (Sweden). Most OECD countries' spend between 20% and 25% of GDP per capita on vocational students. The relationship between government

Figure 2: Income Inequality and Government spending on vocational education



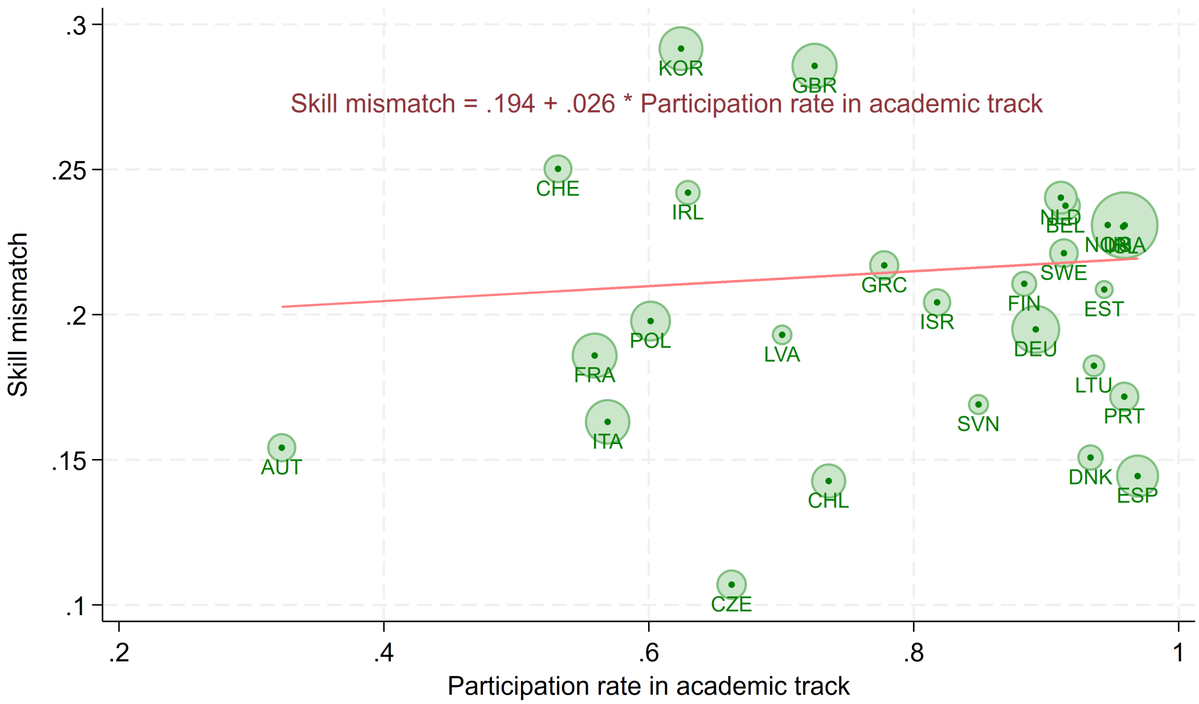
Notes: This figure shows the correlation between income inequality and the government's investment on vocational education in OECD countries. Income inequality is calculated using the 2014-2019 ILOSTAT panel database. The income inequality is the difference in average income levels of service and manufacturing jobs divided by GDP per capita. Government investment on vocational education is from the OECD database in USD per student, and is divided by GDP per capita. We take the average of available annual data by country. The size of the circle reflects the average working population size of the country. We fit a simple regression weighted by working population size, and draw the fitted line.

spending on vocational education and income inequality is overall negative. A simple regression gives a slope of -0.787. This means that if a country increases its spending on each vocational track student increase by 10% of its GDP per capita, the country's income inequality would decrease by 7.9%.

Figure 3 plots the degree of skill mismatch and the academic track participation rate. The degree of skill mismatch ranges between 10.6% (Czech) and 29.2% (Korea). The average degree of mismatch among OECD countries is 20%. It means that about 1/5 of the workers with upper-secondary education or above are either trained in the academic track but get a manufacturing job, or trained in the vocational track but get a service job. Considering the degree of skill mismatch depends on the supply of workers with academic track education and the demand for workers in service jobs, we fit a regression of skill mismatch on the participation rate in academic track and the fraction of service jobs. To illustrate the correlation between skill mismatch and academic track participation, we set the labor demand in service jobs to the average among all OECD countries and draw the fitted line. The slope of -0.026 means that for every 10% increase in academic track participation, the degree of skill mismatch can decrease by 0.26%.

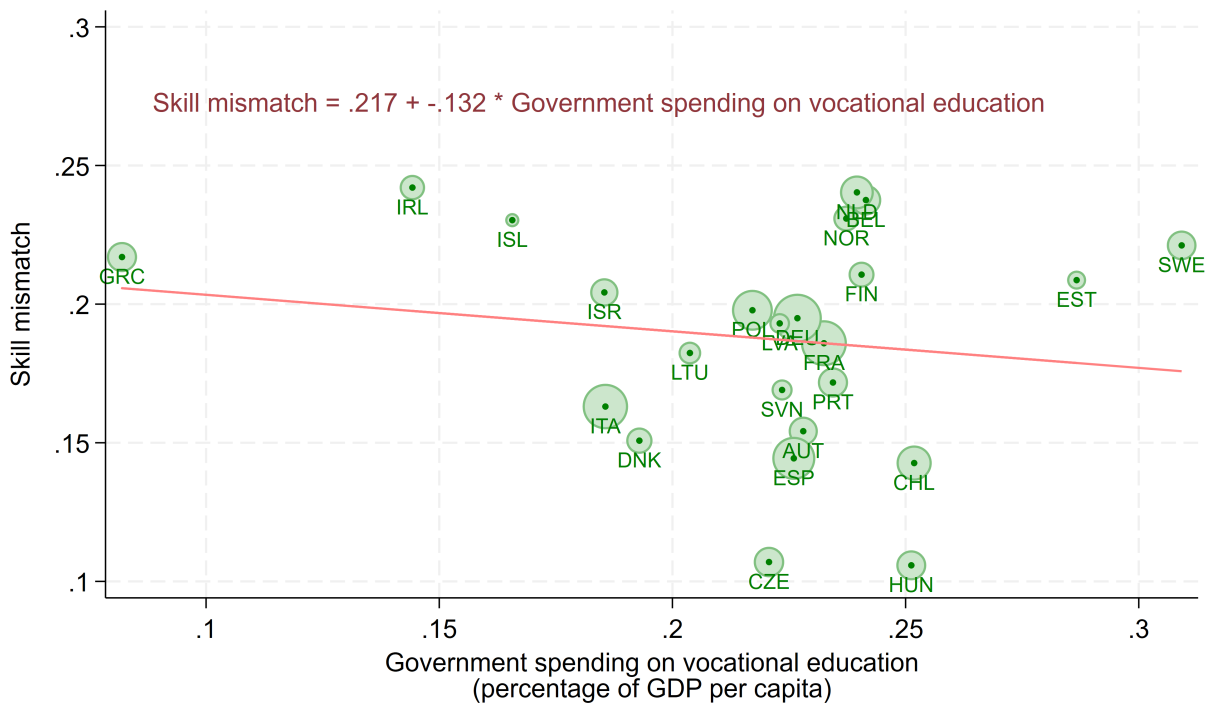
Lastly, in figure 4, we show the negative correlation between government spending on vocational education and the degree of skill mismatch. The fitted regression coefficient indicates that if the government increase the investment in each vocational student by 10% of the GDP per capita, skill mismatch would decrease by 1.32%.

Figure 3: Skill mismatch and Participation Rate in Academic Track



Notes: This figure shows the correlation between the degree of skill mismatch and the government's investment on vocational education in OECD countries. Both statistics are calculated using the 2014-2019 ILOSTAT panel database. The participation rate in academic track is calculated from the total working population and the population of people with vocational training. The degree of skill mismatch is measured by the share of mismatched workers among all working population in service or manufacturing jobs. This variable indicates that their education background and skills demanded by the job are mismatched. Considering the panel data has gaps and is unbalanced, we take the average of available annual data by country. The size of the circle reflects the average working population size of the country. We fit a simple regression weighted by working population size, and draw the fitted line.

Figure 4: Skill mismatch and Government spending on vocational education



Notes: This figure shows the correlation between the degree of skill mismatch and the government's investment on vocational education in OECD countries. The definition of the degree of skill mismatch is the same as Figure 3. The definition of the government investment on vocational education is the same as Figure 2. The size of the circle reflects the average working population size of the country. We fit a simple regression weighted by working population size, and draw the fitted line.

3 The Model

In this section, we provide a simple theoretical framework concerning a social planner's design of an economy's education system.

3.1 The environment

We consider an economy consisting of three types of players: a mass one of individuals, a mass one of job positions/firms, and a social planner who manages the economy's education system which consists of two education tracks: an academic track and a vocational track. In the first stage, the social planner determines the education system; in the second stage (i.e., education sorting), individuals are sorted into the two tracks based upon their ability levels; in the third stage (i.e., labor-market sorting), individuals are selected into job positions based upon their education attainments.

Education: selections and skill development

In the economy, there is a mass one of individuals with a *non-observable* ability level that is uniformly distributed on the support $[0, 1]$. So, an individual with ability level $\theta \in [0, 1]$ is higher ability than θ individuals in the economy. Before entering the labor market, each individual can receive some education, which serves two purposes: one is to select/sort individuals based upon their ability levels, and the other is to develop skills.

There are two different schooling tracks: one $a$cademic, and the other $v$ocational. Individuals are sorted into the two tracks according to their ability levels. That is, in either track an individual with ability θ is admitted only if an individual with ability $\theta' > \theta$ is also admitted, but an individual cannot tell his/her exact ability level based on the admission outcome. The social planner can impose a *quota* $Q \in [0, 1]$ for academic education, i.e., at most Q individuals can receive academic education.

Besides the quota, the social planner can also choose the investment level h (resp. $\tilde{h}$) for vocational (resp. academic) education. We assume that the cost of investment is $s = 0.5ch^2$ (resp. $\tilde{s} = 0.5c\tilde{h}^2$) for each vocational (resp. academic) track student, where $c > 0$ reflects how costly investing in specific skills is for the social planner. Thanks to the investment, an individual can acquire specific skills h (resp. $\tilde{h}$) from vocational (resp. academic) education.[8]

Productivity

There is free entry of firms for the labor market, which consists of two sectors: a $s$ervice sector with a mass of $q \in (0, 1)$ and a $m$anufacture sector with a mass of $1 - q$.[9]

Labor is the only input for production, where service sector productions can better leverage worker ability than manufacture sector productions but manufacture sector productions require some specific skills.[10] Specifically, the output of an arbitrary individual with ability θ and specific skills $l \in \{h, \tilde{h}\}$ is $y_s(\theta, l) = k_s\theta$ as a service-sector worker and $y_m(\theta, l) = k_m\theta + l$ as a manufacture-sector worker, where $0 \leq k_m < k_s$. For ease of

[8] We abstract away from explicit costs (e.g., tuition fees) and implicit costs (e.g., time/efforts) of education, meaning that education by itself is costless for students.

[9] Throughout analyses below, one can also interpret q as a measure of high-paying jobs within an economy.

[10] This is because service-sector productions normally require more comprehensive understanding, critical thinking, interpersonal interactions, and problem solving than manufacture-sector productions, which typically make use of a certain type of industry-specific technical skills. See, for instance, Lucas (1978), Rosen (1982), and Waldman (1984a,b) for related models where a high-level (resp. high-paying) position can better leverage worker ability than a low-level (resp. low-paying) position, and Gibbons and Waldman (2006) and Lazear (2009) for studies with position-specific skills.

exposition, without loss of much generality, we restrict our attentions to equilibrium behavior given $k_m = 0$ and $k_s = 1$. So, the individual's output is given by

$$y_j(\theta, l) = \begin{cases} \theta & \text{if } j = service, \\ l & \text{if } j = manufacture. \end{cases} \quad (1)$$

3.2 The game

The game consists of three main stages.

1. Choice of education systems: Given an economy endowed with $c > 0$ and $0 < q < 1$, the social planner determines the education system $E = \{Q, h; \tilde{h}\}$—Q is the quota for academic education and h (resp. $\tilde{h}$) is the investment level for vocational (resp. academic) education—to maximize the objective function $G_\gamma(Q, h, \tilde{h}|q, c)$ (defined in detail below) in which $\gamma \in (0, 1)$ measures the importance of income inequality.
2. Education sorting: Each individual can apply to both education tracks. Individuals are admitted into academic education based upon their ability levels, subject to the slot constraint that at most Q individuals can be admitted. If an individual is not admitted into academic education, then the individual is admitted into vocational education.
3. Labor-market sorting: Each individual can apply to job positions from both sectors. As education sorting is ability based, firms can infer that an individual's ability level falls into a certain range, but not the exact value. So, an individual's education attainment serves as a signal of the individual's ability à la Spence (1973). Give the productivity specification in (1), applicants are selected into the service sector based upon their education attainments, and selected into the manufacture sector according to their acquired skills. Whenever there are more applicants with an equivalent education attainment than the number of job positions available from a sector, applicants are selected into that sector on a *random* basis.

Given an economy characterized by $c > 0$ and $0 < q < 1$, an equilibrium $\mathcal{E} = \{E; \lambda; W\}$ consists of the social planner's choice of education system $E = \{Q, h, \tilde{h}\}$, the enrollment level λ for academic education, and the wage structure $W = \{w_{ij} | i \in \{a, v\}, j \in \{s, m\}\}$, satisfying:

- Optimality: Each individual's education choice maximizes his/her expected labor-market returns; each individual chooses a job offer with the highest wage.
- Market clearing: Each individual is sorted into one education track during education sorting, and selected into one sector during labor-market sorting.[11]
- Zero profit: Given free entry of firms, an individual receives a job offer equal to his/her expected productivity, making each firm earn a zero expected profit.

We focus on the Perfect Bayesian equilibrium. Table 1 manifests key parameters and variables of the model. When making choices, all players are rational and risk-neutral. The tie-breaking rule during education (resp. labor-market) sorting is that an individual chooses academic education (resp. a service-sector job offer) whenever the two education tracks (resp. job offers) yield an equal expected return.

[11] In equilibrium, each admitted individual enrolls in academic education, and each non-admitted individual enrolls in vocational education which yields a (weakly) higher labor-market return than the choice of receiving no education and entering the labor market immediately. Similarly, each individual (weak) prefers being employed to unemployment.

symbol	meaning
θ	ability level (non-observable)
λ (resp. $1-\lambda$)	enrollment level for academic (resp. vocational) education
Q	quota for academic education
h (resp. $\tilde{h}$)	investment level for vocational (resp. academic) education
$s = \frac{c}{2}h^2$ (resp. $\tilde{s} = \frac{c}{2}\tilde{h}^2$)	investment cost for vocational (resp. academic) education
q (resp. $1-q$)	mass of the service (resp. manufacture) sector
W	wage structure consists of $\{w_{a,s}, w_{a,m}, w_{v,s}, w_{v,m}\}$

Table 1: Summary of notations.

Given an economy endowed with q and c, concerning education sorting, the social planner chooses the investment level h (resp. $\tilde{h}$) for vocational (resp. academic) education and the quota Q for academic education when designing the education system; taking h, $\tilde{h}$, and Q as given, each individual then makes an education choice, which aggregates to the enrollment level λ for academic education which satisfies $\lambda \leq Q$. Regarding labor-market sorting, the wage structure W is jointly determined by q, λ, h, and $\tilde{h}$. Accordingly, we can denote the equilibrium choice of these variables as h^*, $\tilde{h}^*$, Q^*, and λ^*, while the resulting wage structure as W^*.

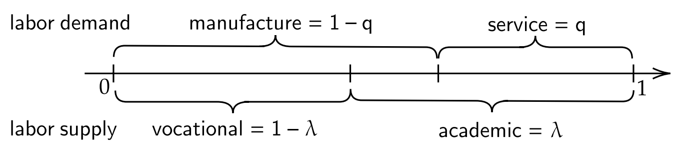


Figure 5: Labor-market sorting when $\Lambda(h) \geq q$.

4 The Analysis

In this section, we analyze the model outlined above for a social planner that not only maximizes the economy's overall output but also minimizes income inequality. To develop model intuitions, we restrict our attentions to equilibrium behavior where there is an excess demand for academic education; also, we assume that the social planner can invest only in vocational education, i.e., $\tilde{h} = 0$, so the education system is now given by $E = \{Q, h\}$. We relax these assumptions in Section 5.

4.1 The equilibrium analysis

Taking the education system $E = \{Q, h\}$ as given, as discussed above, we now study equilibrium behavior when the demand for academic education $\Lambda(h)$ (defined below) satisfies $\Lambda(h) \geq q$.

Returns to education

We first study labor-market returns to the two tracks. Suppose that the equilibrium enrollment level for academic track is λ. If there is no slot constraint for academic education, then $\lambda = \Lambda(h) \geq q$. In this case, as depicted in Figure 5, some academic-track students cannot be employed as service-sector workers

	service	manufacture
academic	$w_{a,s} = 1 - \frac{\lambda}{2}$	$w_{a,m} = 0$
vocational	$w_{v,s} = \frac{1-\lambda}{2}$	$w_{v,m} = h$

Table 2: The wage structure when $\lambda \geq q$.

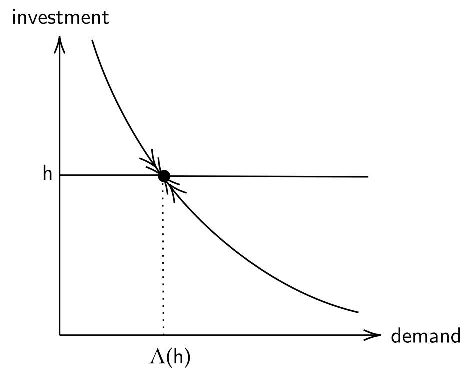


Figure 6: The demand for academic education $\Lambda(h)$.

Given an investment level h, the demand for academic education $\Lambda(h)$ is *stable*. That is, if the demand for academic education is below (resp. above) $\Lambda(h)$, then academic education's return will be higher (resp. lower) than vocational education's, i.e., $q(\frac{1}{\lambda} - \frac{1}{2}) > h$ (resp. $q(\frac{1}{\lambda} - \frac{1}{2}) < h$), which stimulates more demand for academic (resp. vocational) education and, in turn, restores the demand back to $\Lambda(h)$.

and instead become manufacture-sector workers, whereas vocational-track students can all staff service-sector job positions. Given random matching when there are more applicants with an equivalent education attainment than job positions available in a sector, an academic-track student can become a service-sector worker with probability $\frac{q}{\lambda}$.

Due to free entry of firms, the equilibrium wage is equal to a worker's expected ability given the worker's education attainment. As detailed in Table 2, the wage structure is: i) with probability $p_{a,s} = \frac{q}{\lambda}$ (resp. $p_{a,m} = 1 - \frac{q}{\lambda}$), an academic-track student is paid $w_{a,s} = 1 - \frac{\lambda}{2}$ (resp. $w_{a,m} = 0$) when employing as a service-sector (resp. manufacture-sector) worker; and ii) with probability $p_{v,m} = 1$ (resp. $p_{v,s} = 0$), an vocational-track student staffs as a manufacture-sector worker (resp. service-sector worker) and earns $w_{v,m} = h$ (resp. $w_{v,s} = \frac{1-\lambda}{2}$). In turn, the expected labor-market return to academic education and vocational education is, respectively,

$$p_{a,s}w_{a,s} + p_{a,m}w_{a,m} = \frac{q}{\lambda} \cdot \left(1 - \frac{\lambda}{2}\right) + \left(1 - \frac{q}{\lambda}\right) \cdot 0 = q\left(\frac{1}{\lambda} - \frac{1}{2}\right),$$

$$p_{v,s}w_{v,s} + p_{v,m}w_{v,m} = 0 \cdot \frac{1-\lambda}{2} + 1 \cdot h = h.$$

Note that $\partial q\left(\frac{1}{\lambda} - \frac{1}{2}\right)/\partial\lambda < 0$. So, as more individuals receive academic education (i.e., λ increases), the expected labor-market return to vocational track decreases, because of the lower probability $p_{a,s} = \frac{q}{\lambda}$ of entering the service sector and the lower wage $w_{a,s} = 1 - \frac{\lambda}{2}$ as a service-sector worker.

The enrollment level

Using expected returns to the two education tracks derived above, as depicted in Figure 6, we define $\Lambda(h)$ as the enrollment level at which individuals are *indifferent* between the two education tracks, i.e.,

$$q\left(\frac{1}{\Lambda(h)} - \frac{1}{2}\right) = h \Longrightarrow \Lambda(h) = \left(\frac{h}{q} + \frac{1}{2}\right)^{-1}. \quad \text{(IND)}$$

Then, $\Lambda(h)$ captures the demand for academic education, which decreases with the investment level h for vocational track.

Given the demand for academic education $\Lambda(h)$ derived above, we now consider the supply for education. Given the quota Q, there will be a limited supply of academic education. In equilibrium, there are two possible cases, depending on whether or not $Q > \Lambda(h)$. That is,

1. If $Q > \Lambda(h)$ or there is an adequate supply for academic education, then the equilibrium enrollment level for academic education is $\lambda = \Lambda(h) < Q$, where the slot constraint slacks.
2. If $Q \leq \Lambda(h)$ or there is an insufficient supply for academic education, then the equilibrium enrollment level for academic education is $\lambda = Q \leq \Lambda(h)$, where the slot constraint binds.

Note that when there is an insufficient supply for academic education, i.e., $Q < \Lambda(h)$, there is a so-called *education tracking* enforced by the social planner, where at most Q individuals can be admitted into academic education and the other $\Lambda(h) - Q$ individuals are unwillingly separated into vocational education.

In equilibrium, without loss of generality, we can confine our attentions to equilibrium outcomes where the slot constraint binds.

Lemma 1. *Given any $h > 0$, the social planner can focus on $Q \leq \Lambda(h)$ when choosing the quota Q for academic education, suggesting that the equilibrium enrollment level satisfies $\lambda = Q$.*

4.2 The education-system design

With equilibrium results in hand, we then consider the optimal design of education system $E = \{Q, h\}$ from the social planner's perspective. That is, given an economy's endowment, i.e., $0 < q < 1$ as the service sector's size and $c > 0$ as the investment cost, we solve for the social planner's optimal choice of the quota Q for academic education as well as the investment level h for vocational education.

The social planner's objective function

As noted in Section 3, we consider a multi-minded social planner, which aims to not only maximize the economy's overall output but also maintain a low level of income inequality. Define $\gamma \in (0, 1)$ (resp. $1-\gamma$) as the weight or importance of income inequality (resp. overall output) from the social planner's perspective. Then, the social planner's objective function is defined as

$$G_\gamma(Q, h|q, c) = (1-\gamma) \cdot \text{overall output} - \gamma \cdot \text{income inequality} - \text{education spending}.$$

Suppose that the equilibrium enrollment level for academic (resp. vocational) education is λ (resp. $1-\lambda$). Then, the overall output equals

$$\text{overall output} = \underbrace{q \cdot \left(1 - \frac{\lambda}{2}\right)}_{\text{service}} + \underbrace{(\lambda - q) \cdot 0 + (1-\lambda) \cdot h}_{\text{manufacture}} = q\left(1 - \frac{\lambda}{2}\right) + (1-\lambda)\, h;$$

as for income inequality, we assume that the social planner wants to minimize the average wage gap across the two sectors, i.e.,

$$\text{income inequality} = (1-q)\left\{\underbrace{1-\frac{\lambda}{2}}_{\text{service}} - \underbrace{\frac{(\lambda-q)\cdot 0 + (1-\lambda)\cdot h}{1-q}}_{\text{manufacture}}\right\} = (1-q)\left(1-\frac{\lambda}{2}\right) - (1-\lambda)h,$$

which is adjusted (multiplied) by the population size $1-q$ of the low-paying sector (i.e, manufacture).[12,13,14] Using the investment cost (per capita) $s = 0.5ch^2$, the social planner's aggregate spendings on education equal

$$\text{education spending} = (1-\lambda)s = 0.5(1-\lambda)ch^2.$$

Putting together, the social planner's objective function is given by

$$\begin{aligned}(1-\gamma)\left\{q\left(1-\frac{\lambda}{2}\right)+(1-\lambda)\,h\right\} - \gamma\left\{(1-q)\left(1-\frac{\lambda}{2}\right)-(1-\lambda)h\right\} - 0.5(1-\lambda)ch^2 \\ = (q-\gamma)\left(1-\frac{\lambda}{2}\right) + (1-\lambda)\,h - 0.5(1-\lambda)ch^2,\end{aligned}$$

The equity-efficiency tradeoff

We now establish a *conflict* in the design of education system. That is, the social planner faces an *equity-efficiency* tradeoff, because a small quota Q, though increases the overall output, widens the wage gap. Specifically, if the social planner centers on output maximization, i.e., $\gamma \to 0$, it solves

$$\max_{Q \le \Lambda(h), h} G_0(Q, h|q, c) \quad = \quad q\left(1-\frac{Q}{2}\right) + (1-Q)\,h - 0.5(1-Q)ch^2,$$

which decreases in Q. So, the social planner should choose the smallest possible quota, i.e., $Q^* = q$. In turn, the problem reduces to

$$\max_h q\left(1-\frac{q}{2}\right) + (1-q)\,h - 0.5(1-q)ch^2,$$

which yields $h^* = \frac{1}{c}$. In this case, the social planner achieves an *efficient* outcome, i.e., the equilibrium enrollment level is $\lambda = q$, where there is no skill mismatch—each individual with academic (resp. vocational) education is employed as a service (resp. manufacture) sector worker.

[12] As noted in footnote 9, the measure also serves as a proxy for the wage gap between high-paying jobs and low-paying jobs.

[13] Whenever $\Lambda(h) > q$, the average service-sector wage is higher than the average manufacture-sector wage, i.e., $1-\frac{\lambda}{2} > \frac{1-\lambda}{1-q}h$. Specifically, $\Lambda(h) > q$ means $h < 1 - \frac{q}{2}$, which suggests

$$1-\frac{\lambda}{2} - \frac{1-\lambda}{1-q}h > 1 - \frac{\lambda}{2} - \frac{(1-\lambda)(1-\frac{q}{2})}{1-q} = \frac{(1-\frac{\lambda}{2})(1-q) - (1-\lambda)(1-\frac{q}{2})}{1-q} = \frac{\lambda - q}{2(1-q)} > 0.$$

[14] We focus on this specification of income inequality for ease of exposition. Note that results below continue to hold if we use other specifications for the wage gap across the two sectors, e.g., $1-\frac{\lambda}{2} - \frac{1-\lambda}{1-q}h$ or $1-\frac{\lambda}{2} - h$, which all decline with the equilibrium enrollment level λ for academic education. See the appendix for a discussion of alternative measures of income inequality, e.g., variance, Hoover index, Theil index, and Gini coefficient, which are commonly used in practice.

If, to the other extreme, the social planner focuses on inequality minimization, i.e., $\gamma \to 1$, it solves

$$\max_{Q \leq \Lambda(h), h} G_1(Q, h|q, c) = -(1-q)\left(1 - \frac{Q}{2}\right) + (1-Q)\,h - 0.5(1-Q)ch^2,$$

which increases in Q whenever $c(1-q) > 1$.[15] Thus, to reduce income inequality, the social planner chooses the largest possible quota, i.e., $Q^* = \Lambda(h^*)$. In turn, the problem reduces to

$$\max_{h} -(1-q)\left(1 - \frac{\Lambda(h)}{2}\right) + (1-\Lambda(h))\,h - 0.5\,(1-\Lambda(h))\,ch^2,$$

which yields $\Lambda(h^*) = 1$ and, in turn, $h^* = 0$. In this case, due to the focus on equity, the social planner achieves an *inefficient* outcome where there is a large extent of skill mismatches—$1-q$ academic track students lacking specific skills are employed as manufacture-sector workers, and the economy's overall output is, in turn, lower than the efficient counterpart discussed above.

The social planner's problem

The above analysis tells us that, due to the equity-efficiency tradeoff, the social planner tends to choose a quota for academic education higher than the efficient level, i.e., $Q^* = \Lambda(h^*) > q$, given a high importance of equity. Comparing two extreme cases above, though income inequality is reduced, the average wage to the manufacture sector $w_{a,m} = 0$ is now to even lower than the counterpart $w_{v,m} = \frac{1}{c}$ when the social planer solely focuses on efficiency. In fact, as (IND) suggests that the investment level h^* decreases with Q^*, manufacturing workers are made worse off when the quota exceeds the efficient level. That said, in studying the optimal design problem below, we impose a constraint concerning the investment for the social planner. That is, $h \geq \frac{1}{c}$, so the low-paying sector cannot be made worse off when the social planner chooses a large quota due to the equity concern.

Taken together, the social planner solves the following constraint optimization problem when designing the education system:

$$\max_{Q,h}(1-\gamma) = (q-\gamma)\left(1 - \frac{\lambda}{2}\right) + (1-\lambda)\,h - 0.5(1-\lambda)ch^2,$$

subject to the slot constraint $\lambda \leq Q$ as well as the no worse-off constraint $h \geq \frac{1}{c}$. Using Lemma 1, the problem is equivalent to

$$\max_{Q,h}(q-\gamma)\left(1 - \frac{Q}{2}\right) + (1-Q)\,h - 0.5(1-Q)ch^2 \quad \text{s.t.} \quad Q \leq \Lambda(h) \quad \text{and} \quad h \geq \frac{1}{c}. \tag{SP}$$

4.3 The optimal education system

Given an economy endowed with $c > 0$ and $0 < q < 1$, we now characterize the optimal design of education system, i.e., $E^* = \{Q^*, h^*\}$. Given (IND) and the no worse-off constraint $h \geq \frac{1}{c}$, the equilibrium investment level satisfies $h^* = \frac{1}{c}$. Then, the social planner's problem (SP) reduces to

$$\max_{Q \leq \Lambda(h^*)} (q-\gamma)\left(1 - \frac{Q}{2}\right) + \frac{1-Q}{2c}, \tag{2}$$

[15] We elaborate on this condition in more details below.

whose derivative with respect to Q is

$$-\frac{q-\gamma}{2}-\frac{1}{2c}>0 \Longleftrightarrow \gamma>\gamma^*=q+\frac{1}{c},$$

where $0<\gamma^*<1$ given $c(1-q)>1$.

The result below describes the social planner's optimal design of education system.

Proposition 1. *For an economy endowed with $c>0$ and $0<q<1$, assuming $c(1-q)>1$, 1) and 2) below describe the social planner's optimal design of education system $E^*=\{Q^*,h^*\}$.*

1. *The investment level (per capita) is $h^*=\frac{1}{c}$ for vocational track.*
2. *There exists a unique threshold $\gamma^*=q+\frac{1}{c}\in(0,1)$ such that the quota for academic education is $Q^*=q$ if $\gamma\le\gamma^*$, and $Q^*=\Lambda(h^*)>q$ otherwise, where $\Lambda(h^*)=\left(\frac{1}{2}+\frac{1}{cq}\right)^{-1}$. In particular, when there is an expansion for academic education, i.e., $Q^*>q$, the degree of expansion $\frac{\Lambda(h^*)}{q}=\left(\frac{q}{2}+\frac{1}{c}\right)^{-1}$ satisfies $\partial\frac{\Lambda(h^*)}{q}/\partial c>0$ and $\partial\frac{\Lambda(h^*)}{q}/\partial q<0$.*

This result tells us that there exists a threshold value γ^* concerning the importance of inequality reduction. That is, the social planner imposes a quota $Q=\Lambda(h^*)>q$ whenever $\gamma>\gamma^*$. This equilibrium behavior can be envisaged as an *expansion* of academic education, where the social planner chooses an inefficiently high quota. In this case, an inefficiently large number of individuals can pursue academic education, which results in an equilibrium outcome with *diploma inflation*: academic-track students find it more difficult to stand out with their education attainments and, in turn, receive a lower labor-market return.

Our theory rationalizes why it is of the social planner's interest to expand academic education. That is, typically a more natural approach to reduce income inequality is for the social planner to increase the investment in vocational education, which increases the wage to the manufacture sector and, in turn, reduces the wage gap across the two sectors. However, when the investment cost c is too high, i.e., $c(1-q)>1$ discussed above, the wage to the manufacture sector $h^*=\frac{1}{c}$ is relatively too low, which requires the social planner to take further actions to reduce income inequality. According to our analysis, a large quota $Q^*=\Lambda(h^*)>q$ for academic education or an expansion of academic education, though entails diploma inflation, serves to decrease the wage to the service sector $1-\frac{Q^*}{2}$ which, in turn, narrows the wage gap across the two sectors.

This result also tells us that the threshold γ^* decreases in c and increases in q, meaning that an economy endowed with a high c and/or a low q is more likely to expand academic expansion in order to reduce inequality. The intuition is that if c is high, the investment level $h^*=\frac{1}{c}$ is relatively low; thus, as discussed above, the social planner can expand academic education by choosing an inefficiently large quota $Q^*=\Lambda(h^*)>q$ which reduces the wage gap across the two sectors. Moreover, $\partial\frac{\Lambda(h^*)}{q}/\partial c>0$ and $\partial\frac{\Lambda(h^*)}{q}/\partial q<0$ mean that an economy with a larger c and/or a smaller q tends to expand academic education to a greater extent. By contrast, if c is low, the investment level $h^*=\frac{1}{c}$ is already high, which suggests a relatively small wage gap across the two sectors; consequently, the social planner can focus on output maximization and, in turn, chooses an efficient quota $Q^*=q$, where there is no expansion for academic education.

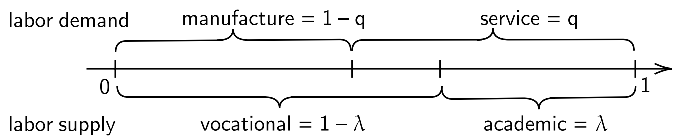


Figure 7: Labor-market sorting when $\Lambda(h) \leq q$.

	service	manufacture
academic	$w_{a,s} = 1 - \frac{\lambda}{2}$	$w_{a,m} = 0$
vocational	$w_{v,s} = \frac{1-\lambda}{2}$	$w_{a,m} = h$

Table 3: The wage structure when $\lambda \leq q$.

5 Extensions

In this section, we present three enriched analyses. To better understand the essence of equilibrium behavior, we twist the baseline model in one direction at a time. That is, 1) we study the equilibrium with $\Lambda(h) \leq q$, where there is an inverted wage gap across the two sectors; 2) we consider refined education sorting, where academic-track students can be further sorted into two tiers according to their ability levels; and 3) we study two types of education-system reforms: i) whether to integrate the two education tracks, where the social planner can invest in both vocational track and academic track; and ii) when to conduct education sorting, where the social planner additionally decides when to sort students into the two tracks.

5.1 An inverted wage gap

In this section, we analyze equilibrium behavior when the demand for academic education satisfies $\Lambda(h) \leq q$, and show that it leads to an inverted wage gap in equilibrium, i.e., the average wage to the manufacture sector exceeds that to the service sector.

The equilibrium characterization

When $\Lambda(h) \leq q$, the demand for academic education is less than q, meaning that the equilibrium enrollment level for academic education is as high as q, i.e., $\Lambda(h) \leq q \Rightarrow \lambda \leq q$. In turn, there must be more vocational-track students than the number of job positions available in the manufacture sector, i.e., $1 - \lambda \geq 1 - q$. In this case, as depicted in Figure 7, some vocational-track students cannot be employed as manufacture-sector workers and instead become service-sector workers, whereas academic-track students can all staff service-sector job positions. Given random matching when there are more applicants with an equivalent education attainment than job positions available in a sector, an vocational-track student can become a manufacture-sector worker with probability $\frac{1-q}{1-\lambda}$.

Though the matching probability differs from the baseline analysis, the wage structure is still as described by Table 2. That is: i) with probability $p_{a,s} = 1$ (resp. $p_{a,m} = 0$), an academic-track student is paid $w_{a,s} = 1 - \frac{\lambda}{2}$ (resp. $w_{a,m} = 0$) when employing as a service-sector (resp. manufacture-sector) worker; and ii) with probability $p_{v,m} = \frac{1-q}{1-\lambda}$ (resp. $p_{v,s} = 1 - \frac{1-q}{1-\lambda}$), an vocational-track student staffs as a manufacture-sector (resp. service-sector) worker and earns $w_{v,m} = h$ (resp. $w_{v,s} = \frac{1-\lambda}{2}$). In turn, the expected labor-market return to academic track and vocational track is, respectively,

$$p_{a,s}w_{a,s} + p_{a,m}w_{a,m} = 1 \cdot \left(1 - \frac{\lambda}{2}\right) + 0 \cdot 0 = 1 - \frac{\lambda}{2},$$
$$p_{v,s}w_{v,s} + p_{v,m}w_{v,m} = \left(1 - \frac{1-q}{1-\lambda}\right) \cdot \frac{1-\lambda}{2} + \frac{1-q}{1-\lambda} \cdot h.$$

Analogous to (IND) from the baseline analysis, the demand for academic education $\Lambda(h)$ makes individuals indifferent between the two education tracks, i.e.,

$$1 - \frac{\lambda}{2} = \left(1 - \frac{1-q}{1-\lambda}\right) \frac{1-\lambda}{2} + \frac{1-q}{1-\lambda} h \Longleftrightarrow \Lambda(h) = 1 - (1-q)\left(1 - \frac{q}{2}\right)^{-1} h, \qquad \text{(IND')}$$

where $\Lambda(h)$ decreases in h.

The optimal education system

Suppose that λ is the equilibrium enrollment level for academic education, where $\lambda \leq q$ given $\Lambda(h) \leq q$. Then, the economy's overall output is

$$\underbrace{\lambda\left(1 - \frac{\lambda}{2}\right) + (q-\lambda)\frac{1-\lambda}{2}}_{\text{service}} + \underbrace{(1-q)h}_{\text{manufacture}} = \frac{q}{2} + (1-q)\left(\frac{\lambda}{2} + h\right),$$

while income inequality is

$$q\left\{\underbrace{h}_{\text{manufacture}} - \underbrace{\frac{\lambda \cdot \left(1 - \frac{\lambda}{2}\right) + (q-\lambda) \cdot \frac{1-\lambda}{2}}{q}}_{\text{service}}\right\} = qh - \frac{q + (1-q)\lambda}{2},$$

which is adjusted (multiplied) by the affected population size q in the low-paying sector (i.e., service).[16] Then, given $\gamma \in (0,1)$ as the importance of income inequality, the social planner's objective function is given by

$$(1-\gamma)\left\{\frac{q}{2} + \left(\frac{\lambda}{2} + h\right)(1-q)\right\} - \gamma\left\{qh - \frac{q + (1-q)\lambda}{2}\right\} - \frac{(1-\lambda)ch^2}{2},$$

which increases in λ. Thus, as opposed to the baseline analysis, the social planner does *not* face an equity-efficiency tradeoff when $\Lambda(h) \leq q$.

We now characterize the optimal design of education system. As discussed above, $\Lambda(h) \leq q$ suggests more vocational-track students than the number of job positions available in the manufacture sector, i.e., $1 - \lambda \geq 1 - q$, the social planner now, instead, imposes a quota for vocational track, which is denoted as $\tilde{Q}$. By setting $\tilde{Q} = 1 - q$, the social planner ensures that the enrollment level for academic education is $\lambda = q$.

[16] In contrast to the baseline analysis, when $\Lambda(h) < q$, there is an *inverted* wage gap across the two sectors where the wage to the manufacture sector is on average higher than that to the service sector, because $\Lambda(h) < q$ suggests $\lambda < q$ and, in turn,

$$\frac{\lambda\left(1 - \frac{\lambda}{2}\right) + (q-\lambda)\frac{1-\lambda}{2}}{q} = \frac{1}{2} + \frac{\lambda}{2}\left(\frac{1}{q} - 1\right) < \frac{1}{2} + \frac{q}{2}\left(\frac{1}{q} - 1\right) = 1 - \frac{q}{2} \leq h.$$

In turn, the optimization problem is

$$\max_h (1-\gamma)\left\{\frac{q}{2}+\left(\frac{q}{2}+h\right)(1-q)\right\}-\gamma\left\{qh-\frac{q+(1-q)q}{2}\right\}-\frac{(1-q)ch^2}{2},$$

which yields $h^*=\max\left\{\left(1-\frac{\gamma}{1-q}\right)\frac{1}{c},1-\frac{q}{2}\right\}$.[17]

The result below describes the social planner's optimal choice of education system when $\Lambda(h)\leq q$.

Proposition 2. *For an economy endowed with $c>0$ and $0<q<1$, 1) and 2) below describe the social planner's optimal design of education system $E^*=\{\tilde{Q}^*,h^*\}$.*

1. *The investment level (per capita) is $h^*=\max\left\{\left(1-\frac{\gamma}{1-q}\right)\frac{1}{c},1-\frac{q}{2}\right\}$ for vocational track.*
2. *Regardless of the value of γ, the quota for vocational education is always $\tilde{Q}^*=1-q$.*

Given $\tilde{Q}^*=1-q$ as the quota for vocational education, even if $\left(1-\frac{\gamma}{1-q}\right)\frac{1}{c}>1-\frac{q}{2}$, the equilibrium enrollment level for vocational education is bounded above by $1-q$; in turn, the equilibrium enrollment level for academic education is $\lambda=q$, where there is no skill mismatch.

5.2 Refined education sorting

In the baseline analysis, each academic track student employed by the service sector receives the same wage in equilibrium, because there is just one type of schooling outcome within academic education. So, information content of academic education is *coarse*. In this section, we enrich the baseline analysis with a *refined* sorting of academic students, where there can be multiple schooling outcomes within academic education. For instance, outstanding students (i.e., those with high ability levels) can be admitted into elite schools or further pursue advanced degrees.

The equilibrium characterization

In the analysis below, we maintain the assumption that individuals are sorted into academic education based on their ability levels, but the sorting is refined in the sense that the most capable λ_1 individuals can now stand out and receive tier 1 academic education which distinguishes themselves from others who receive tier 2 academic education. In turn, the sorting of academic track students to service-sector positions is no longer fully random, in that tier 1 students are now more likely to be employed by the service sector and receive a higher wage than tier 2 students.

For ease of exposition, we assume λ_1 is exogenous and satisfies $\lambda_1<q$. Also, we define $\Lambda(h)$ as the demand for tier 1 and tier 2 academic education and Q as the corresponding quota. The analysis below focuses on equilibrium behavior when $\Lambda(h)\geq q$, i.e., there are more tier 1 and tier 2 students in total than there are service-sector positions. Then, as depicted in Figure (8), tier 1 students can all be employed as service-sector workers, whereas some tier 2 students cannot be employed as service-sector workers and instead become manufacture-sector workers. As was true for the baseline analysis, random matching suggests that a tier 2 student can become a service-sector worker with probability $\frac{q-\lambda_1}{\lambda-\lambda_1}\in(0,1)$.

As detailed in Table 4, the wage structure is: i) with probability $p_{a_1,s}=1$ (resp. $p_{a_1,m}=0$), a tier 1 student is paid $w_{a_1,s}=1-\frac{\lambda_1}{2}$ (resp. $w_{a_1,m}=0$) as a service-sector (resp. manufacture-sector) worker; ii) with probability $p_{a_2,s}=\frac{q-\lambda_1}{\lambda-\lambda_1}$ (resp. $p_{a_2,m}=1-\frac{q-\lambda_1}{\lambda-\lambda_1}$), a tier 2 student is paid $w_{a_2,s}=1-\lambda_1-\frac{\lambda-\lambda_1}{2}$

[17] Specifically, $\Lambda(h)\leq q$ is equivalent to $h\geq 1-\frac{q}{2}$. See the appendix for more details.

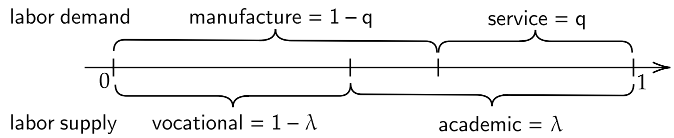


Figure 8: Labor-market sorting with a refined education sorting.

	service	manufacture
tier 1 academic	$w_{a_1,s} = 1 - \frac{\lambda_1}{2}$	$w_{a_1,m} = 0$
tier 2 academic	$w_{a_2,s} = 1 - \lambda_1 - \frac{\lambda - \lambda_1}{2}$	$w_{a_2,m} = 0$
vocational	$w_{v,s} = \frac{1-\lambda}{2}$	$w_{v,m} = h$

Table 4: The wage structure with refined education sorting.

(resp. $w_{a_2,m} = 0$) as a service-sector (resp. manufacture-sector) worker; and iii) with probability $p_{v,m} = 1$ (resp. $p_{v,s} = 0$), an vocational student is paid $w_{v,m} = h$ (resp. $w_{v,s} = \frac{1-\lambda}{2}$) as a manufacture-sector (resp. service-sector) worker. Like for the baseline analysis, the demand for academic education $\Lambda(h)$ equates the two tracks' labor-market returns, i.e.,

$$\frac{q}{\Lambda(h)}\left(1 - \frac{\Lambda(h)}{2}\right) + \frac{\lambda_1}{2}\left(1 - \frac{q}{\Lambda(h)}\right) = h \Longrightarrow \Lambda(h) = \left(1 - \frac{\lambda_1}{2}\right)\left(\frac{h}{q} + \frac{1}{2} - \frac{\lambda_1}{2q}\right)^{-1}. \tag{IND-R}$$

with $\partial\Lambda(h)/\partial\lambda_1 > 0$.[18]

The optimal education system

Suppose that λ is the equilibrium enrollment level of academic education (including tier 1 and tier 2). Then, the economy's overall output is

$$\underbrace{\lambda_1 \cdot \left(1 - \frac{\lambda_1}{2}\right) + (q - \lambda_1) \cdot \left(1 - \lambda_1 - \frac{\lambda - \lambda_1}{2}\right)}_{\text{service}} + \underbrace{(\lambda - q) \cdot 0 + (1 - \lambda)\, h}_{\text{manufacture}}$$
$$= \lambda_1\left(1 - \frac{\lambda_1}{2}\right) + (q - \lambda_1)\left(1 - \frac{\lambda + \lambda_1}{2}\right) + (1 - \lambda)\, h,$$

[18] Specifically, the labor-market return to academic education is

$$\underbrace{\frac{\lambda_1}{\lambda}\left(1 - \frac{\lambda_1}{2}\right)}_{\text{tier 1}} + \frac{\lambda - \lambda_1}{\lambda}\underbrace{\left\{\frac{q - \lambda_1}{\lambda - \lambda_1} \cdot \left(1 - \lambda_1 - \frac{\lambda - \lambda_1}{2}\right) + \left(1 - \frac{q - \lambda_1}{\lambda - \lambda_1}\right) \cdot 0\right\}}_{\text{tier 2}}$$
$$= \frac{\lambda_1}{\lambda}\left(1 - \frac{\lambda_1}{2}\right) + \frac{q}{\lambda}\left(1 - \frac{\lambda + \lambda_1}{2}\right) - \frac{\lambda_1}{\lambda}\left(1 - \frac{\lambda + \lambda_1}{2}\right) = \frac{q}{\lambda}\left(1 - \frac{\lambda}{2}\right) + \frac{\lambda_1}{2}\left(1 - \frac{q}{\lambda}\right),$$

which increases in λ_1, because an academic student can potentially stand out and be sorted into tier 1. Thus, as more individuals can stand out given refined education sorting, there is an increased demand for academic education.

while income inequality is

$$(1-q)\Bigg\{\underbrace{\frac{\lambda_1\cdot(1-\frac{\lambda_1}{2})+(q-\lambda_1)\cdot(1-\lambda_1-\frac{\lambda-\lambda_1}{2})}{q}}_{\text{service}}-\underbrace{\frac{(\lambda-q)\cdot 0+(1-\lambda)\,h}{1-q}}_{\text{manufacture}}\Bigg\}$$
$$=(1-q)\frac{\lambda_1}{q}\left(1-\frac{\lambda_1}{2}\right)+\left(1-\frac{\lambda_1}{q}\right)\left(1-\frac{\lambda+\lambda_1}{2}\right)-(1-\lambda)h.$$

In turn, the social planner's objective function is given by

$$(1-\gamma)\left\{\lambda_1\left(1-\frac{\lambda_1}{2}\right)+(q-\lambda_1)\left(1-\frac{\lambda+\lambda_1}{2}\right)+(1-\lambda)\,h\right\}$$
$$-\gamma\left\{(1-q)\left\{\frac{\lambda_1}{q}\left(1-\frac{\lambda_1}{2}\right)+\left(1-\frac{\lambda_1}{q}\right)\left(1-\frac{\lambda+\lambda_1}{2}\right)\right\}-(1-\lambda)h\right\}-\frac{(1-\lambda)ch^2}{2},$$

where the output (i.e., efficiency) decreases in λ whereas income inequality (i.e., equity) increases in λ. So, the argument from the baseline analysis continues to hold given refined sorting, i.e., the social planner again faces an equity-efficiency tradeoff when choosing the quota Q, where $\lambda \le Q$. In particular, if $\gamma \to 0$, the social planner chooses $Q^* = q$ and $h^* = \frac{1}{c}$.[19] So, the no worse-off constraint is $h \ge \frac{1}{c}$. Using Lemma 1, the social planner solves the following constrained optimization problem:

$$\max_{Q\le\Lambda(h),h\ge\frac{1}{c}}(1-\gamma)\left\{\lambda_1\left(1-\frac{\lambda_1}{2}\right)+(q-\lambda_1)\left(1-\frac{Q+\lambda_1}{2}\right)+(1-Q)\,h\right\}$$
$$-\gamma\left\{(1-q)\left\{\frac{\lambda_1}{q}\left(1-\frac{\lambda_1}{2}\right)+\left(1-\frac{\lambda_1}{q}\right)\left(1-\frac{Q+\lambda_1}{2}\right)\right\}-(1-Q)h\right\}-\frac{(1-Q)ch^2}{2}.$$

We now characterize the optimal design of education system. Given (IND-R) and the no worse-off constraint $h \ge \frac{1}{c}$, the equilibrium investment level satisfies $h^* = \frac{1}{c}$. Then, the social planner's problem becomes

$$\max_{Q\le\Lambda(h^*)}(1-\gamma)\left\{\lambda_1\left(1-\frac{\lambda_1}{2}\right)+(q-\lambda_1)\left(1-\frac{Q+\lambda_1}{2}\right)\right\}$$
$$-\gamma(1-q)\left\{\frac{\lambda_1}{q}\left(1-\frac{\lambda_1}{2}\right)+\left(1-\frac{\lambda_1}{q}\right)\left(1-\frac{Q+\lambda_1}{2}\right)\right\}+\frac{1-Q}{2c},$$

whose derivative with respect to Q is

$$-\frac{(1-\gamma)\,(q-\lambda_1)}{2}+\frac{\gamma(1-q)}{2}\left(1-\frac{\lambda_1}{q}\right)-\frac{1}{2c}>0\Longleftrightarrow\gamma>\gamma_R^*=q+\frac{1}{c\left(1-\frac{\lambda_1}{q}\right)},$$

where $0<\gamma_R^*<1$ given $c(1-q)\left(1-\frac{\lambda_1}{q}\right)>1$.

Given refined education sorting, the result below describes the social planner's optimal design of education system.

Proposition 3. *For an economy endowed with $c > 0$ and $0 < q < 1$, assuming $c(1-q)\left(1-\frac{\lambda_1}{q}\right) > 1$, 1)*

[19] This investment level remains the same as that from the baseline analysis, because refined education sorting has no effect on the productivity of manufacture-sector workers.

	service	manufacture
academic	$w_{a,s} = 1 - \frac{\lambda}{2}$	$w_{a,m} = \tilde{h}$
vocational	$w_{v,s} = \frac{1-\lambda}{2}$	$w_{v,m} = h$

Table 5: The wage structure with integrated tracks.

and 2) below describe the social planner's optimal design of education system $E^* = \{Q^*, h^* | \lambda_1\}$.

1. *The investment level (per capita) is $h^* = \frac{1}{c}$ for vocational track.*
2. *There exists a unique threshold $\gamma_R^* = q + \frac{1}{c(1-\frac{\lambda_1}{q})} \in (0,1)$ such that the quota for academic track is $Q^* = q$ if $\gamma \leq \gamma_R^*$, and $Q^* = \Lambda(h^*) > q$ otherwise, where $\Lambda(h^*) = \left(1 - \frac{\lambda_1}{2}\right)\left(\frac{1}{cq} + \frac{1}{2} - \frac{\lambda_1}{2q}\right)^{-1}$. In particular, when there is an expansion for academic track, i.e., $Q^* > q$, the degree of expansion $\frac{\Lambda(h^*)}{q} = \left(1 - \frac{\lambda_1}{2}\right)\left(\frac{1}{c} + \frac{q}{2} - \frac{\lambda_1}{2}\right)^{-1}$ satisfies $\partial\frac{\Lambda(h^*)}{q}/\partial c > 0$, $\partial\frac{\Lambda(h^*)}{q}/\partial q < 0$, and $\partial\frac{\Lambda(h^*)}{q}/\partial\lambda_1 > 0$.*

Basically, this results extends the logic behind Proposition (1) to an education system with refined sorting of academic students. That is, the social planner can use an inefficiently high quota to reduce income inequality. $\partial\gamma_R^*/\partial\lambda_1 > 0$ suggests that the social planner is now less likely to expand academic education. The logic is that, as was true for the baseline analysis, expanding academic education can still reduce the wage gap, yet the expansion only decreases the wage to tier 2 students but not to tier 1 students. However, whenever there is an expansion of academic education, $\partial\frac{\Lambda(h^*)}{q}/\partial\lambda_1 > 0$ suggests that the degree of expansion is higher than that from the baseline analysis.

5.3 Education-system reforms

5.3.1 Whether to integrate education tracks

In the baseline analysis, we assume that the social planner can invest *only* in vocational education. In this section, we enrich the baseline analysis by integrating two education tracks, where the social planner can invest in *both* tracks, i.e., $h > 0$ for vocational education and $\tilde{h} > 0$ for academic education. As a consequence, students from either track can acquire some specific skills, which we call as integrated tracks because teaching contents/materials across the two tracks are more aligned for developing skills specific to the manufacture sector.

The equilibrium characterization

As described in Table 5, the wage structure and the matching probability are basically the same as those from the baseline analysis, except for the wage $w_{a,m} = \tilde{h}$ to an academic-track student employed as a manufacture-sector worker because the social planner can now also invest $\tilde{h}$ in academic education. As for the baseline analysis, we can define $\Lambda(h, \tilde{h})$ as the demand for academic education, which equates the two tracks' labor-market returns, i.e.,

$$\frac{q}{\lambda}\left(1 - \frac{\lambda}{2}\right) + \left(1 - \frac{q}{\lambda}\right)\tilde{h} = h \Longrightarrow \Lambda(h, \tilde{h}) = \left(1 - \tilde{h}\right)\left(\frac{h}{q} + \frac{1}{2} - \frac{\tilde{h}}{q}\right)^{-1}, \qquad \text{(IND-I)}$$

where $\partial\Lambda(h, \tilde{h})/\partial h < 0$ and $\partial\Lambda(h, \tilde{h})/\partial\tilde{h} > 0$. So, integrated tracks where $\tilde{h} > 0$ suggest an increased return to academic education and, in turn, a higher demand for academic education than separated tracks where

$\tilde{h} = 0$, i.e., $\Lambda(h, \tilde{h}) > \Lambda(h, 0) = \left(\frac{h}{q} + \frac{1}{2}\right)^{-1}$ for any $\tilde{h} > 0$.[20]

The optimal education system

Given integrated tracks, the overall output and income inequality are, respectively, given by

$$\underbrace{q\left(1 - \frac{\lambda}{2}\right)}_{\text{service}} + \underbrace{(\lambda - q)\tilde{h} + (1 - \lambda)\, h}_{\text{manufacture}} \quad \text{and} \quad (1 - q)\left\{\underbrace{1 - \frac{\lambda}{2}}_{\text{service}} - \underbrace{\frac{(\lambda - q)\tilde{h} + (1 - \lambda)h}{1 - q}}_{\text{manufacture}}\right\}$$

Then, the social planner's objective function is equal to

$$(1 - \gamma)\left\{q\left(1 - \frac{\lambda}{2}\right) + (\lambda - q)\tilde{h} + (1 - \lambda)h\right\} - \gamma\left\{(1 - q)\left(1 - \frac{\lambda}{2}\right) - (\lambda - q)\tilde{h} - (1 - \lambda)h\right\}$$
$$- \frac{\lambda c\tilde{h}^2 + (1 - \lambda)ch^2}{2} = (q - \gamma)\left(1 - \frac{\lambda}{2}\right) + (\lambda - q)\tilde{h} + (1 - \lambda)h - \frac{\lambda c\tilde{h}^2 + (1 - \lambda)ch^2}{2},$$

where the output (i.e., efficiency) decreases in λ whereas income inequality (i.e., equity) increases in λ. Thus, similar to the baseline analysis, the social planner again faces an equity-efficiency tradeoff when choosing the quota Q, where $\lambda \leq Q$. In particular, if $\gamma \to 0$, the social planner chooses $Q^* = q$, $h^* = \frac{1}{c}$, and $\tilde{h}^* = 0$. Then, if $\gamma > 0$, the corresponding no worse-off constraints are $h \geq \frac{1}{c}$ and $\tilde{h}^* \geq 0$ to ensure that the low-paying sector cannot be made worse off when the social planner chooses a large quota due to the equity concern. Using Lemma 1, the social planner solves the following constrained optimization problem:

$$\max_{Q \leq \Lambda(h,\tilde{h}), h \geq \frac{1}{c}, \tilde{h} \geq 0} (q - \gamma)\left(1 - \frac{Q}{2}\right) + (Q - q)\tilde{h} + (1 - Q)h - \frac{Qc\tilde{h}^2 + (1 - Q)ch^2}{2}.$$

We now characterize the optimal design of education system. Whenever inequality reduction requires an expansion of academic education, i.e., $Q^* > q$, the social planner chooses the largest possible quota subject to no worse-off constraints $h \geq \frac{1}{c}$ and $\tilde{h}^* \geq 0$. As (IND-I) suggests $\partial\Lambda(h, \tilde{h})/\partial h < 0$ and $\partial\Lambda(h, \tilde{h})/\partial\tilde{h} > 0$, the social planner chooses $h^* = \frac{1}{c}$, and $\tilde{h}^* = \left(1 - \frac{q}{Q}\right)\frac{1}{c}$.[21] In turn, the social planner's problem becomes

$$\max_{Q \leq \Lambda(h,\tilde{h})} (q - \gamma)\left(1 - \frac{Q}{2}\right) + \frac{1 - Q}{2c} + \frac{Q}{2c}\left(1 - \frac{q}{Q}\right)^2 = (q - \gamma)\left(1 - \frac{Q}{2}\right) + \frac{1}{2c}\left(1 + \frac{q^2}{Q} - 2q\right), \tag{3}$$

whose derivative with respect to Q is

$$-\frac{(1 - \gamma)q}{2} + \frac{\gamma(1 - q)}{2} - \frac{1}{2c}\frac{q^2}{Q^2} > 0 \iff \gamma > \gamma_I^* = q + \frac{q^2}{cQ^2},$$

where $0 < \gamma_I^* < 1$ given $c(1 - q)Q^2/q^2 \geq c(1 - q) > 1$.

Given integrated tracks, the result below describes the social planner's optimal design of education system.

[20] Specifically, $\Lambda(h, \tilde{h}) = \left(1 - \tilde{h}\right)\left(\frac{h}{q} + \frac{1}{2} - \frac{\tilde{h}}{q}\right)^{-1} > q$ suggests that $h + \frac{q}{2} < 1$ and, in turn, $\frac{h}{q} + \frac{1}{2} < \frac{1}{q}$.

[21] Given integrated tracks, the social planner invests less heavily in academic track than in vocational track, i.e., $\tilde{h}^* = \left(1 - \frac{q}{Q}\right)\frac{1}{c} < \frac{1}{c} = h^*$ which increases in q and decreases in Q, because the investment in academic track applies to Q students but only $Q - q$ of them can be employed in the manufacture sector and, in turn, benefit from the investment.

Proposition 4. *For an economy endowed with $c > 0$ and $0 < q < 1$, assuming $c(1 - q) > 1$, 1) and 2) describe the social planner's optimal design of education system $E^* = \{Q^*, h^*, \tilde{h}^*\}$.*

1. *The investment level (per capita) is $h^* = \frac{1}{c}$ for vocational track and $\tilde{h}^* = \left(1 - \frac{q}{Q^*}\right)\frac{1}{c}$ for academic track.*
2. *There exists a unique threshold $\gamma_I^* = q + \frac{q^2}{c(\Lambda(h^*,\tilde{h}^*))^2} \in (0, 1)$ such that the quota for academic track is $Q^* = q$ if $\gamma \leq \gamma_I^*$, and $Q^* = \Lambda(h^*, \tilde{h}^*) > q$ otherwise, where $\Lambda(h^*, \tilde{h}^*)$ is uniquely determined by*

$$\Lambda(h^*, \tilde{h}^*) = \left(1 - \frac{1}{c} + \frac{q}{c\Lambda(h^*, \tilde{h}^*)}\right)\left(\frac{1}{2} + \frac{1}{c\Lambda(h^*, \tilde{h}^*)}\right)^{-1}.$$

   In particular, when there is an expansion for academic track, i.e., $Q^* > q$, the degree of expansion $\frac{\Lambda(h^*,\tilde{h}^*)}{q} = \left(1 - \frac{1}{c} + \frac{q}{cQ^*}\right)\left(\frac{q}{2} + \frac{q}{cQ^*}\right)^{-1}$ satisfies $\partial\frac{\Lambda(h^*,\tilde{h}^*)}{q}/\partial c > 0$ and $\partial\frac{\Lambda(h^*,\tilde{h}^*)}{q}/\partial q < 0$.[22]

As $\Lambda(h, \tilde{h}) > \Lambda(h, 0) > q$ for any $\tilde{h} > 0$, we have $\gamma_I^* < \gamma^*$, which suggests that an economy with integrated tracks is more likely to expand academic education than an economy with segregated tracks. The logic is that integrated tracks result in a smaller degree of skill mismatches—academic students who fail to be employed by the service sector can still generate an output of $\tilde{h} > 0$ in the manufacture sector. Hence, while decreasing the wage gap, the associated efficiency loss due to skill mismatches is smaller given integrated tracks than given segregated tracks; this provides the social planner with more opportunities to reduce income inequality via expanding academic education. Thus, the degree of expansion given integrated tracks is greater than that given segregated tracks, i.e., $\frac{\Lambda(h^*,\tilde{h}^*)}{q} > \frac{\Lambda(h^*,0)}{q} > q$.

By comparing the social planner's value functions given separated tracks and given integrated tracks, we find that (3) outweighs (2) whenever $1 + \frac{q^2}{Q^*} - 2q > 1 - Q^*$ or $\frac{q^2}{Q^*} - 2q + Q^* = \frac{1}{Q^*}(Q^* - q)^2 > 0$. Thus, the social planner has incentives to integrate the two education tracks whenever the status quo of the economy's education system is associated with an expansion of academic education, i.e., the quota for academic education satisfies $Q^* > q$. In this case, integrated tracks result in a lesser extent of skill mismatches and a smaller degree of income inequality than separated tracks.

Corollary 1. *Whenever the status quo features an expansion of academic education, i.e., $Q^* > q$, the social planner has incentives to integrate the two tracks.*

5.3.2 When to conduct education tracking?

In the baseline model, our analysis is agnostic to when should individuals be sorted into academic education and vocational education—the timing of education tracking, which is an important choice variable of the social planner in the real world. In this section, we incorporate the timing of education sorting into the design of education system.

The equilibrium characterization

When designing an economy's education system, we now assume that the social planner determines a triple $\{Q, h, \alpha\}$, in which Q and h are as defined in the baseline analysis and α reflects the timing of education tracking—the specific stage of a student's career at which students are sorted into the two education tracks.

[22] Specifically, $Q^* = \Lambda(h^*, \tilde{h}^*) > q$ is unique because it solves $\frac{Q^*}{2} + \frac{1}{c} = 1 - \frac{1}{c} + \frac{q}{cQ^*}$, where $\frac{Q^*}{2} + \frac{1}{c}$ increases in Q^*, $1 - \frac{1}{c} + \frac{q}{cQ^*}$ decreases in Q^*, and $\frac{Q^*}{2} + \frac{1}{c} = \frac{q}{2} + \frac{1}{c} < 1 = 1 - \frac{1}{c} + \frac{q}{cq}$ when $Q^* = q$.

	service	manufacture
academic	$w_{a,s} = \left(1-\frac{\lambda}{2}\right)\sqrt{\alpha} + \frac{1}{2}\left(1-\sqrt{\alpha}\right)$	$w_{a,m} = 0$
vocational	$w_{v,s} = \frac{1-\lambda}{2}\sqrt{\alpha} + \frac{1}{2}\left(1-\sqrt{\alpha}\right)$	$w_{v,m} = h\sqrt{1-\alpha}$

Table 6: The wage structure with an endogenous timing choice of education tracking.

Concerning the timing choice for education tracking α, the social planner now faces a tradeoff. That is, if α increases or education tracking is deferred to a later stage of a student's career, the sorting will be more based upon an individual's ability; in turn, an individual's schooling choice serves as a stronger signal of the individual's ability. In particular, $\alpha = 1$ suggests that education tracking is completely ability based which coincides with the baseline case, whereas $\alpha = 0$ means that education tracking is fully random. On the other hand, as α increases, there is a declining marginal return to investing in vocational education, because there will be less time left for students to acquire specific skills during vocational education when education tracking is deferred to a later stage of a student's career.

Specifically, as described in Table 6, the wage to academic track students on service-sector positions is equal to $w_{a,s} = \left(1-\frac{\lambda}{2}\right)\sqrt{\alpha} + \frac{1}{2}\left(1-\sqrt{\alpha}\right) = \frac{1}{2} + \frac{1-\lambda}{2}\sqrt{\alpha}$, i.e., a weighted average of two extreme-case wages $1-\frac{\lambda}{2}$ (when education tracking is completely ability based) and $\frac{1}{2}$ (when education tracking is purely random), where the weight $\sqrt{\alpha}$ reflects that the extent to which education tracking is ability based increases in α at a declining rate; the wage to vocational track students on manufacture-sector positions is given by $w_{v,m} = h\sqrt{1-\alpha}$. Like for the baseline analysis, we define $\lambda(h;\alpha)$ as the demand for academic education which equates the two tracks' labor-market returns, i.e.,

$$\frac{q}{\lambda}\cdot\left(\frac{1}{2}+\frac{1-\lambda}{2}\sqrt{\alpha}\right) + \left(1-\frac{q}{\lambda}\right)\cdot 0 = q\left(\frac{1+\sqrt{\alpha}}{2\lambda} - \frac{\sqrt{\alpha}}{2}\right) = h\sqrt{1-\alpha}$$

$$\Longrightarrow \Lambda(h;\alpha) = \frac{1+\sqrt{\alpha}}{2}\left(\frac{\sqrt{1-\alpha}}{q}h + \frac{\sqrt{\alpha}}{2}\right)^{-1}, \quad \text{(IND-S)}$$

where $\Lambda(h;\alpha)$ increases in α. This is intuitive because the return to academic education increases with α due to the increased accuracy of ability sorting, whereas the return to vocational education declines with α due to the less time left for skill development. Though α can equal to zero in theory, typically α cannot be too small in the real-world design of education system. For ease of exposition, we assume for the analysis below that there exists a lower bound $\hat{\alpha}$ for α, where $\hat{\alpha}$ satisfies $c(1-q)\sqrt{\hat{\alpha}} > 1-\hat{\alpha}$.[23]

The optimal education system

Similar to the baseline analysis, the social planner's objective function is given by

$$(1-\gamma)\left\{q\left(\frac{1}{2}+\frac{1-\lambda}{2}\sqrt{\alpha}\right) + (1-\lambda)\,h\sqrt{1-\alpha}\right\} - \gamma(1-q)\left\{\frac{1}{2}+\frac{1-\lambda}{2}\sqrt{\alpha} - \frac{1-\lambda}{1-q}h\sqrt{1-\alpha}\right\}$$
$$-0.5(1-\lambda)ch^2 = -(\gamma-q)\left(\frac{1}{2}+\frac{1-\lambda}{2}\sqrt{\alpha}\right) + (1-\lambda)\,h\sqrt{1-\alpha} - 0.5(1-\lambda)ch^2.$$

As was true for the baseline analysis, the social planner faces an equity-efficiency tradeoff when choosing the quota Q, where $\lambda \leq Q$. In particular, if $\gamma \to 0$, the social planner chooses $Q^* = q$ and $h^* = \frac{\sqrt{1-\alpha}}{c}$. So, the

[23] One justification for this assumption is that a small α means that students are sorted into the two tracks at a relatively early stage of student career. In this case, education tracking is largely non-ability based. In turn, teaching will be less effective, because schools need to tailor teaching contents/materials to students with a dispersed distribution of ability levels.

no worse-off constraint is $h \geq \frac{\sqrt{1-\alpha}}{c}$. Using Lemma 1, the social planner solves the following constrained optimization problem:

$$\max_{Q \leq \Lambda(h;\alpha), h \geq \frac{\sqrt{1-\alpha}}{c}, \alpha \in [0,1]} -(\gamma - q)\left(\frac{1}{2} + \frac{1-Q}{2}\sqrt{\alpha}\right) + (1-Q)\, h\sqrt{1-\alpha} - 0.5(1-Q)ch^2.$$

We now characterize the optimal design of education system. Given (IND-S) and the no worse-off constraint $h \geq \frac{\sqrt{1-\alpha}}{c}$, the equilibrium investment level satisfies $h^* = \frac{\sqrt{1-\alpha}}{c}$. Then, the social planner's problem becomes

$$\max_{Q \leq \Lambda(h;\alpha), \alpha \in [0,1]} -(\gamma - q)\left(\frac{1}{2} + \frac{1-Q}{2}\sqrt{\alpha}\right) + \frac{1-Q}{2c}(1-\alpha).$$

Concerning the optimal timing of education sorting, if $\gamma \geq q$, then $\alpha^* = \hat{\alpha}$; if $\gamma < q$, the first-order condition with respect to α yields

$$\alpha^* = \frac{c^2(q-\gamma)^2}{4} \in [\hat{\alpha}, 1].$$

As for the optimal quota, the derivative with respect to Q is

$$\frac{\gamma - q}{2}\sqrt{\alpha^*} - \frac{1-\alpha^*}{2c} > 0 \Longleftrightarrow \gamma > \gamma_S^* = q + \frac{1-\alpha^*}{c\alpha^*},$$

where $0 < \gamma_S^* < 1$ given $\alpha^* \geq \sqrt{\alpha}$ where $c(1-q)\sqrt{\hat{\alpha}} > 1 - \hat{\alpha}$.

For ease of exposition, below we describe the social planner's optimal design of education system.

Proposition 5. *For an economy endowed with $c > 0$ and $0 < q < 1$, assuming $c(1-q)\sqrt{\hat{\alpha}} > 1 - \hat{\alpha}$, 1) through 3) below describe the social planner's optimal design of education system $E^* = \{Q^*, h^*, \alpha^*\}$.*

1. *The timing of education sorting is $\alpha^* = \max\left\{\frac{c^2(q-\gamma)^2}{4}, \hat{\alpha}\right\}$ if $\gamma < q$ and $\alpha^* = \hat{\alpha}$ otherwise, where $\frac{c^2(\gamma-q)^2}{4}$ decreases with γ and increases with c and q.*
2. *The investment level (per capita) is $h^* = \frac{\sqrt{1-\alpha^*}}{c}$ for vocational track.*
3. *There exists a unique threshold $\gamma_S^* = q + \frac{1-\alpha^*}{c\sqrt{\alpha^*}} \in (0,1)$ such that the quota for academic track is $Q^* = q$ if $\gamma \leq \gamma_S^*$ and $Q^* = \Lambda(h^*;\alpha^*) > q$ otherwise, where $\Lambda(h^*;\alpha^*) = \frac{1+\sqrt{\alpha^*}}{2}\left(\frac{1-\alpha^*}{qc} + \frac{\sqrt{\alpha^*}}{2}\right)^{-1}$. In particular, when there is an expansion for academic track, i.e., $Q^* > q$, the degree of expansion $\frac{\Lambda(h^*;\alpha^*)}{q} = \frac{1+\sqrt{\alpha^*}}{2}\left(\frac{1-\alpha^*}{c} + \frac{q\sqrt{\alpha^*}}{2}\right)^{-1}$ satisfies $\partial\frac{\Lambda(h^*;\alpha^*)}{q}/\partial c > 0$ and $\partial\frac{\Lambda(h^*;\alpha^*)}{q}/\partial q < 0$.*

This result tells us what happens if the social planner can manipulate the investment level, the quota for academic education, and the timing of education tracking. When $\gamma \geq q$, i.e., equity is important and/or the service sector is small, we find an early sorting $\alpha^* = \hat{\alpha}$. In particular, the social planner imposes a large quota $Q^* > q$ if $\gamma > \gamma_S^*$ and a small quota $Q^* = q$ if $q \leq \gamma \leq \gamma_S^*$. The logic is that given a large γ and/or a small q, the social planner should pay more attention to equity when designing the education system. As an early sorting of student can decrease the extent to which education tracking is ability based and increase the rate of return to investment, a smaller α not only decreases the wage to the service sector but also increases the wage to the manufacture sector; in turn, the social planner achieves a smaller degree of income inequality.[24]

[24]There is, however, a policy debate that an early sorting condemns students placed into vocational education.

By contrast, if $\gamma < q$, i.e., efficiency is important and/or the service sector is large, we find a small quota $Q^* = q$ as well as a delayed sorting $\alpha^* > \hat{\alpha}$, where $\alpha^* = \frac{c^2(q-\gamma)^2}{4}$ increases in q and decreases in γ, i.e., a country with a small γ and/or a large q defers the sorting to a later stage of student career. The logic is that given a small γ and/or a large q, the social planner should pay more attention to efficiency. Thus, by deferring the sorting of student, the social planner increases the extent to which education tracking is ability based and, in turn, increases the overall output.

6 Implications

Based on equilibrium results above, we now elaborate on the relevance of our theoretical analysis to empirical facts described in Section 2. Also, we discuss issues related to the design and reform of education system.

6.1 Relevance to the data

We relate equilibrium results above to stylized facts concerning cross-country differences in education systems, skill mismatches, and income inequality. In doing so, we first describe economic variables of interests in terms of our model elements. That is, 1) the investment level (per capita) in vocational education: $INV = h^* = \frac{1}{c}$; 2) the degree of expansion for academic education: $EXP = \frac{\lambda}{q}$, i.e., the ratio of academic track students to the mass of the service sector; 3) the degree of skill mismatches: $SMM = \frac{\lambda - q}{1-q}$, i.e., the proportion of manufacture sector workers who are academic track students;[25] 4) the degree of income inequality: $IIQ = (1-q)\left(1 - \frac{\lambda}{2} - \frac{1-\lambda}{1-q}h^*\right) = (1-q)\left(1 - \frac{\lambda}{2}\right) - (1-\lambda)h^*$.

Consider an economy whose education system exhibits an expansion for academic education. If the social planner increases the investment in vocational education, in line with empirical findings in Section 2, we find a smaller degree of expansion for academic education, a smaller degree of skill matches, and a smaller degree of income inequality.[26]

Corollary 2. *Given an expansion of academic education, i.e., the social planner chooses a large quota, it holds in equilibrium that $\partial EXP/\partial INV < 0$, $\partial SMM/\partial INV < 0$, and $\partial IIQ/\partial INV < 0$.*

expanding academic education can reduce income inequality. This result depends on two important factors: the cost of educational investments, represented by the value of c, and the availability of high-paying jobs, represented by q. Educational investments are costly, as highlighted by data from the OECD, which indicates that reskilling an economy requires significant resources. Additionally, good job positions are relatively rare, as shown by a low value of q. The model examines two potential quota choices: one where $Q = q$, representing no expansion of academic education, and one where $Q > q$, representing expansion. These choices correspond to different types of economies. Economies with a low c and/or a high q, such as Austria, the Czech Republic, Germany, and Nordic countries, are characterized by high investment in vocational education and a small academic quota, resulting in a high participation rate in vocational education. In contrast, economies with a high c and/or a low q, such as China, Japan, and the USA, invest less in vocational education and have a larger academic quota, leading to a lower participation rate in vocational training. When a country expands academic education to reduce income inequality, as seen in those with

[25] An alternative measure of skill mismatches is $\frac{\lambda - q}{\lambda} = 1 - \frac{q}{\lambda}$, i.e., the proportion of academic track students employed by the manufacture sector.

[26] If the education system exhibits a high level of investments in vocational education, the labor market is featured with no expansion of academic education (i.e., $EXP = \frac{\lambda}{q} = 1$) which is independent of h^*, no skill mismatch (i.e., $SMM = \frac{\lambda - q}{1-q} = 0$) which is independent of h^*, and income inequality $(1-q)\left(1 - \frac{q}{2}\right) - (1-q)h^* = (1-q)\left(1 - \frac{q}{2} - h^*\right)$ which decreases in h^*.

well-developed higher education systems (where λ_1 is large), tier 1 students receive significantly higher wages than tier 2 students, creating a large wage gap across sectors. This also leads to lower labor-market returns for tier 2 students, resulting in more severe diploma inflation for lower-tier academic education.

6.2 Policy recommendations

The policy implications derived from the model suggest several key areas of focus for improving education systems. First, there is a need for increased investment in vocational education, though this can be quite costly. For instance, while more than 55% of higher education admissions in China are in vocational programs, these programs receive only about 20% of the total financial investment in higher education. This imbalance highlights the need for greater financial commitment. In addition to increasing the amount of investment, it is crucial to improve the effectiveness and quality of vocational education investments. In countries with more developed vocational education systems, such as Austria and Germany, strong partnerships between enterprises and schools have been established. This collaboration ensures that vocational training aligns closely with the actual needs of firms, helping to reduce skill mismatches and improve labor market outcomes.

Another important policy implication is improving the participation rate in vocational education by making it more attractive to students. This can be achieved through public campaigns, offering more pathways to career advancement, and ensuring that vocational education provides students with relevant and high-quality skills. Additionally, it is essential to break the social prejudice that vocational education is "inferior" or carries a “stigma” in the labor market. Overcoming this bias will require efforts to promote the value of vocational education as a viable and respected path to stable and well-paying jobs, similar to the perception of academic education.

6.3 Education-system reforms

Our theory provides valuable insights into potential education-system reforms, particularly the integration of academic and vocational tracks and the timing of education sorting.

Integrated Tracks An integrated education system broadens the channels for talent development by offering various pathways for individuals to acquire necessary skills, knowledge, and experience. In this model, secondary vocational education (the vocational track) and regular high school education (the academic track) are combined. Both tracks collaborate to design courses, exchange teaching staff, and implement credit recognition and student status transfers. This integrated approach is an essential aspect of education reform as it meets the needs of individual growth while addressing the diverse developmental requirements of traditional high schools.

In the U.S., integrated education systems often feature large quotas for academic education ($Q > q$) and minimal tracking, as seen in the use of co-taught course curriculums (Betts (2011)). Although formal tracking is reduced, there is still within-school ability grouping through elective courses and advanced placement programs, offering customized teaching based on student abilities. By contrast, countries like Austria and Germany implement an integrated system with smaller quotas for academic education ($Q = q$), focusing on the close alignment of vocational and academic education to better match labor market needs.

Optimal Timing of Education Sorting The optimal timing of when students are sorted into academic and vocational tracks is another critical aspect of education-system reform. In Nordic countries, such as

Denmark and Norway, recent reforms have introduced de-tracking policies. These reforms expand the academic track and defer the sorting of students into tracks until a later stage in their education. De-tracking provides more equality of opportunity by allowing students more time to develop before making career-defining choices. However, this delay in sorting can reduce teaching efficiency by limiting the time available for vocational education, which is crucial for developing industry-specific skills.

Moreover, studies such as Dustmann, Puhani, and Schönberg (2017) suggest that education reforms must also address the correction of initial sorting outcomes. Delayed sorting can offer opportunities to revise or adjust early educational placements, ensuring that students are placed in tracks that best suit their abilities and interests, thereby improving long-term labor market outcomes.

These reforms demonstrate the importance of tailoring education systems to both the demands of the labor market and the goal of reducing inequality, balancing the need for vocational specialization with broader access to academic opportunities.

7 Conclusion

This paper investigates the optimal design of an economy's education system, focusing on how education serves both to develop specific skills for students and to sort them into academic and vocational tracks based on ability. Using data from OECD countries, we examine the relationship between different education systems and labor market outcomes, including income inequality and skill mismatches. Our model provides a framework in which a social planner designs an education system that balances economic output with minimizing inequality. The baseline model illustrates how an expansion of academic education can lead to increased skill mismatches and diploma inflation, while also potentially reducing income inequality. We explore several extensions, including the effects of wage gaps, refined education sorting, and education-system reforms. These extensions show how altering the design of education systems can impact labor market outcomes, particularly in countries with different levels of investment in vocational education. Our findings highlight the critical trade-offs policymakers face when designing education systems that aim to optimize both efficiency and equity. Lastly, we draw out policy implications for improving vocational education investment, increasing participation rates, and overcoming the stigma associated with vocational training. The results are relevant for both developed and developing economies looking to align education systems more closely with labor market demands while addressing social inequalities.

While this paper contributes to our understanding of optimal education system design, several avenues for future research remain unexplored. One important area involves accounting for firm heterogeneity. In our model, job positions within the service sector are treated as homogeneous, but it would be valuable to investigate the impact of heterogeneity in firm productivity within both the service and manufacturing sectors. For example, our findings on tier 1 students being over-educated in the service sector could change if firms were ranked by productivity, allowing more skilled students to match with higher-paying firms, potentially altering the education-labor market dynamic.

Another extension is to account for unemployment in the model. Currently, we assume all workers find employment, but in reality, labor markets are often characterized by mismatches where there are fewer firms than workers, leading to unemployment. Examining how unemployment affects education system design could provide insights into how education and labor markets interact under more constrained conditions.

Additionally, future research could explore the design of school admissions. Our model focuses on ability-based student selection but does not consider how effort or investment in tutoring affects admissions. For example, in countries with merit-based admissions, tutoring may reduce randomness and increase the accuracy of ability sorting, but banning tutoring could introduce more randomness into the sorting process, which would in turn influence wage gaps.[27] It would be interesting to explore how different admissions criteria affect the relationship between education and labor market outcomes, particularly in systems where admissions are less strictly ability-based.

Lastly, future research could examine other policies aimed at reducing inequality, such as affirmative action or wage regulations. While expanding academic education can reduce inequality, policies like wage ceilings for high-income earners or wage floors for low-income workers may complement educational reforms. However, such policies introduce new challenges, including tax avoidance and evasion, and the behavioral responses of individuals, which could limit the social planner's ability to redistribute income effectively.[28]

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[27] See Lee and Suen (2023) for a study on manipulating test scores via tutoring (i.e., student investments) during school admissions, where a student's test score is determined by the student's innate ability and spending on tutoring.

[28] In this case, the social planner faces some new issues: 1) tax avoidance (legal means to reduce tax liability) or evasion (illegal under-reporting of income); 2) individuals' behavioral response might limit the social planner's ability to redistribute with taxes/transfers.

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A Technical Details

A.1 The empirical analysis

A.2 The theoretical analysis

A.2.1 Three equilibrium regimes

Recall from (IND') that the demand for academic education, which decreases with the investment level h in vocational education, is given by

$$\Lambda(h) = 1 - (1-q)\left(1 - \frac{q}{2}\right)^{-1} h,$$

from where we obtain $\Lambda(h) = 0$ if $h = \left(1 - \frac{q}{2}\right)(1-q)^{-1}$ and $\Lambda(h) = q$ if $h = 1 - \frac{q}{2}$. Then, there are three possible equilibrium regimes.

1. If the investment level is low, i.e., $h \leq 1 - \frac{q}{2}$, the demand for academic education satisfies $\Lambda(h) \geq q$, where $\Lambda(h) = \left(\frac{h}{q} + \frac{1}{2}\right)^{-1}$. This case corresponds to the baseline analysis in Section 4, where $\Lambda(h) \geq q$ amounts to $h \leq 1 - \frac{q}{2}$ or $c\left(1 - \frac{q}{2}\right) \geq 1$.
2. If the investment level is in the intermediate range, i.e., $1 - \frac{q}{2} \leq h < \left(1 - \frac{q}{2}\right)(1-q)^{-1}$, the demand for academic education satisfies $0 < \Lambda(h) \leq q$, where $\Lambda(h) = 1-(1-q)\left(1 - \frac{q}{2}\right)^{-1} h$. This case corresponds to the enriched analysis in Section 5.1, where $\Lambda(h) \leq q$ amounts to $h \geq 1 - \frac{q}{2}$ or $c\left(1 - \frac{q}{2}\right) \leq 1$.
3. If the investment level is high, i.e., $h \geq \left(1 - \frac{q}{2}\right)(1-q)^{-1}$, the demand for academic education satisfies $\Lambda(h) = 0$. We do not consider this case in our analysis.

A.2.2 Relevance to the data

In equilibrium, it is obvious that the investment level $h^* = \frac{1}{c}$ decreases in c and the enrollment level $\lambda = \Lambda(h^*) = \left(\frac{h^*}{q} + \frac{1}{2}\right)^{-1} = \left(\frac{1}{qc} + \frac{1}{2}\right)^{-1}$ increases in c. In turn, we obtain $\partial\Lambda(h^*)/\partial h^* < 0$, which suggests: i) the degree of expansion for academic education $EXP = \frac{\Lambda(h^*)}{q}$ decreases in h^*; and ii) the degree of skill mismatches $SMM = \frac{\Lambda(h^*)-q}{1-q}$ increases in $\Lambda(h^*)$ and decreases in h^*. Using $h^* = q\left(\frac{1}{\Lambda(h^*)} - \frac{1}{2}\right)$ from (IND), the degree of income inequality is given by

$$IIQ = (1-q)\left(1 - \frac{\Lambda(h^*)}{2}\right) - (1 - \Lambda(h^*))\, q\left(\frac{1}{\Lambda(h^*)} - \frac{1}{2}\right) = 1 + \frac{q}{2} - \frac{\Lambda(h^*)}{2} - \frac{q}{\Lambda(h^*)},$$

which increases in $\Lambda(h^*)$ and decreases in h^*.[1]

[1] Specifically, $\frac{\partial IIQ}{\partial \Lambda(h^*)} = -\frac{1}{2} + \frac{q}{(\Lambda(h^*))^2} > 0$ is implied by $\Lambda(h^*) = \left(\frac{1}{cq} + \frac{1}{2}\right)^{-1} < \sqrt{2q}$, which holds whenever $0 < q < 0.5$ is sufficiently small to ensure that $c\left(\sqrt{\frac{q}{2}} - \frac{q}{2}\right) < 1$.

B Web Appendix (Not Intended for Publication)

B.1 Generic distributions of worker ability

Given $y_m(\theta, 0) = 0$, the economy's overall output and income inequality are, respectively, given by

$$\frac{q}{1-F(1-\lambda)}\int_{1-\lambda}^{1} y_s(\theta,0)\,dF(\theta) + \int_{0}^{1-\lambda} y_m(\theta,h)\,dF(\theta) = \frac{q}{1-F(1-\lambda)}\int_{1-\lambda}^{1}\theta dF(\theta) + F(1-\lambda)h,$$

$$\frac{1}{1-F(1-\lambda)}\int_{1-\lambda}^{1} y_s(\theta,0)\,dF(\theta) - \frac{1}{1-q}\int_{0}^{1-\lambda} y_m(\theta,h)\,dF(\theta) = \frac{1}{1-F(1-\lambda)}\int_{1-\lambda}^{1}\theta dF(\theta) - \frac{F(1-\lambda)h}{1-q}.$$

Using Lemma 1 from the main text, the social planner's problem is equivalent to

$$\max_{Q\le\Lambda(h)}(1-\gamma)\left\{\frac{q}{1-F(1-Q)}\int_{1-Q}^{1}\theta dF(\theta) + F(1-Q)h\right\} - \gamma\left\{\frac{1-q}{1-F(1-Q)}\int_{1-Q}^{1}\theta dF(\theta) - F(1-Q)h\right\} - 0.5F(1-Q)ch^2,$$

where $\Lambda(h)$ denotes the enrollment level at which individuals are indifferent between the two tracks, i.e.,

$$\frac{q}{\Lambda(h)}\frac{1}{1-F(1-\Lambda(h))}\int_{1-\Lambda(h)}^{1}\theta dF(\theta) = h.$$

By choosing $h^* = \frac{1}{c}$, the above problem reduces to

$$\max_{Q\le\Lambda(h)}\frac{q-\gamma}{1-F(1-Q)}\int_{1-Q}^{1}\theta dF(\theta) + \frac{F(1-Q)}{2c}.$$

When the expected ability level of academic track graduates declines with the enrollment level, i.e.,

$$\mathcal{D} := \frac{\partial}{\partial Q}\left\{\frac{1}{1-F(1-Q)}\int_{1-Q}^{1}\theta dF(\theta)\right\} < 0,$$

the first-order condition with respect to Q yields

$$(q-\gamma)\mathcal{D} - \frac{f(1-Q)}{2c} > 0 \Longleftrightarrow \gamma > \gamma^* = q + \frac{f(1-Q)}{-2c\mathcal{D}}.$$

In turn, $Q^* = q$ if $\gamma \le \gamma^*$ and $Q^* = \Lambda(h^*) > q$ otherwise.

B.2 Alternative measures of income inequality

In the main text, we use the wage difference across the service sector and the manufacture sector as a measure for income inequality, which serves as a proxy for various measures of income inequality that emphasize on earning gaps between high-paying positions (resp. the rich) and low-paying positions (resp. the poor), e.g., S80/S20, P90/P10, P90/P50, Palma ratio, etc.

In this section, we examine alternative measures for income inequality, including the income variance, the Hoover index, the Theil index, the Gini coefficient, which are commonly used in practice.[2] Below, we

[2] Let x_i be the income of the i-th person and $\bar{x}$ be the mean income. Then, as detailed below, the variance, the Hoover

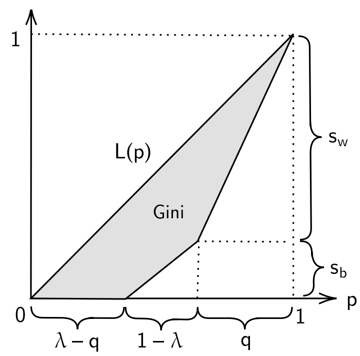

Figure 9: The Gini coefficient.

examine whether, as described in the main text, there is an equity-efficiency tradeoff when the social planner design its education system.

Define $\mu = q\left(1-\frac{Q}{2}\right) + (1-Q)h$. The **variance** is given by

$$q\left(1-\frac{Q}{2}-\mu\right)^2 + (Q-q)\left(0-\mu\right)^2 + (1-Q)(\mu-h)^2$$

which can be shown to decline with Q given a sufficiently small h (which means that $c > 0$ is sufficiently large). In particular, if $c = +\infty$, we have $h = 0$ and $\mu = q\left(1-\frac{Q}{2}\right)$. In turn, the variance is equal to $q(1-q)^2\left(1-\frac{Q}{2}\right)^2 + (1-q)q^2\left(1-\frac{Q}{2}\right)^2$, which decreases in Q. Hence, there is an equity-efficiency tradeoff. The **Hoover** index is given by

$$\frac{1}{2}\frac{q\left|1-\frac{Q}{2}-\mu\right| + (Q-q)\left|0-\mu\right| + (1-Q)\left|h-\mu\right|}{q\left(1-\frac{Q}{2}\right)+(1-Q)h},$$

which increases in Q. Hence, there exists no equity-efficiency tradeoff. The **Theil** index is given by

$$q\frac{1-\frac{Q}{2}}{\mu}\ln\left(\frac{1-\frac{Q}{2}}{\mu}\right) + (1-Q)\frac{h}{\mu}\ln\left(\frac{h}{\mu}\right),$$

which increases in Q. Hence, there exists no equity-efficiency tradeoff. The **Gini** coefficient is defined as

$$2 \times \text{area below the } 45^\circ \text{ line and above the Lorenz curve,}$$

where the Lorenz curve $L(p)$ tracks the fraction of total income earned by individuals below each percentile p. In particular, a Gini coefficient of 0 means perfect equality, while a Gini coefficient of 1 means complete inequality—the top income earners owns all the income. As depicted in Figure 9, the aggregated income

index, and the Theil index are, respectively, defined as $\sigma^2 = \sum_i (x_i - \bar{x})^2$, $H = \frac{1}{2}\frac{\sum_i |x_i - \bar{x}|}{\sum_i x_i}$, and $T = \sum_i \frac{x_i}{\bar{x}} \ln\left(\frac{x_i}{\bar{x}}\right)$, where a higher means that a higher degree of income inequality (dispersions) within an economy.

share of q service-sector workers who earn $1-\frac{Q}{2}$ is

$$s_w := \frac{q\left(1-\frac{Q}{2}\right)}{q\left(1-\frac{Q}{2}\right)+(1-Q)\,h} = \frac{q}{q+\frac{1-Q}{1-\frac{Q}{2}}h},$$

which increases in Q, while the aggregated income share of $1-Q$ manufacture-sector workers who earn h is $s_b := 1-s_w$. In turn, the area below the Lorentz curve is given by

$$\frac{qs_w}{2}+qs_b+\frac{(1-Q)s_b}{2} = \frac{1}{2}\left[qs_w+2q(1-s_w)+(1-Q)(1-s_w)\right] = \frac{1}{2}\left[q+(1-Q+q)(1-s_w)\right],$$

which decreases in Q. Thus, to minimize the Gini coefficient, a small quota $Q=q$ is chosen to maximize the area below the Lorentz curve. Hence, there is not an equity-efficiency tradeoff.